ملزمة 1

Surveying

1^{st} year civil

Notes (1)

2012/2013

C.S.C

Civil Students committee

Theory of Errors

Any Survey process include certain part of Errors due to various Factors, therefore the true value is impossible.

So We Get the Most Probable value (MPV).

Sources of errors:

1) Natural errors: Due to atmospheric or surrounding enviroment.

Ex: Effect of temp., pressure, Refraction ------

2) Instrumental error: Due to used surveying Instrument

Ex: Missing part of used tape, Index error -----

3) Personal error: Due to surveyor or observer.

Ex: Reading error, plummeting of instrument ----

* Kinds (types) of errors :

1) Mistakes (Blunders) : Personal usually

* Should be detected and eliminated.

2) Systematic error : Personal - Instrumental - Natural.

* It comulates with the same sign.

* Treated by computed corrections and adopting certain observing techniques. or calibration if instrumental.

3) Random error : Personal - Instrumental - Natural.

* Different values for the same observed quantity.

* Treated by using the theory of error (decrease its affect)

Random errors Characteristics:

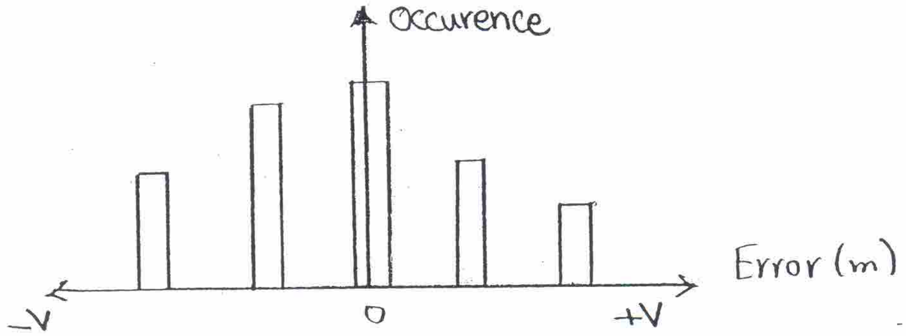
1) Probability of -ve errors = Probability of +ve errors
2) Probability of large errors is small, while probability of small errors is large
3) Follow Gauss probability distribution function

Gauss Probability Normal Distribution Function (NDF)

$$MPV(\bar{L}) = \frac{L_1 + L_2 + L_3 + \text{--------} + L_n}{n} = \frac{\sum_{i=1}^{i=n} L}{n} \quad \text{Arithmetic Mean}$$

$$\text{Residual } (V) = L_i - \bar{L}$$

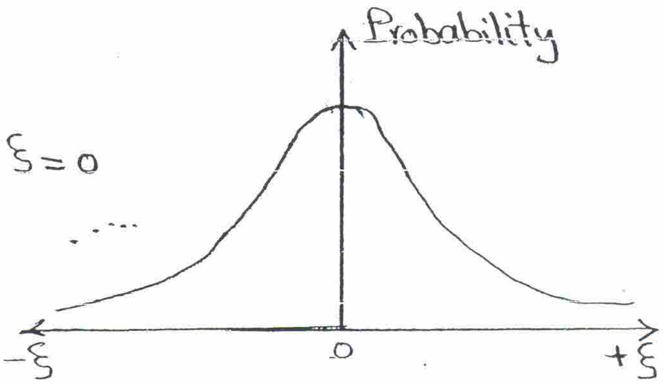
If we Plot the residuals with the number of occurence of each residual, we get Histogram of errors



If no. of observations (n) is very large $\Longrightarrow n = \infty$

MPV becomes the true Value L^* $\Longrightarrow V = \text{True error } (\xi)$

Histogram $\Longrightarrow$ NDF



Properties of NDF

a) Bell Shaped
b) Maximum at $\xi = 0$
c) Area under Curve = 1
d) Parallel to ξ-axis at $\xi = \pm\infty$
e) Symmetric around $\xi = 0$

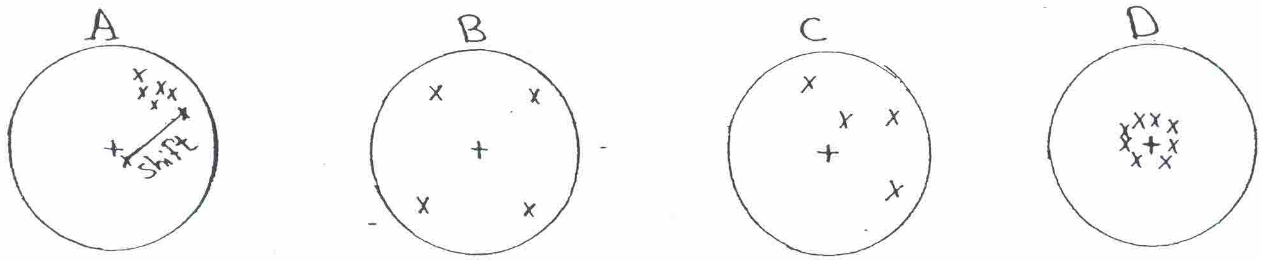
* Measure of rondom errors *

. Precision

* Degree of closeness between one observation of a single quantity & the other observations of the same quantity

. Accuracy

* Degree of closeness between each single observation of a single quantity & its true value.



A	B	C	D
Precise & not accurate (systematic errors)	Accurate but not precise	Not accurate nor precise	Accurate & precise

* يتم الحكم على precission عن طريق ~

* لا نستطيع الحكم على Accurracy حيث لا نعرف القيمه الحقيقيه "True value"

* Measure of Precision :

a) Average Error (A) $= \dfrac{\sum_{i=1}^{i=n} |V_i|}{n}$

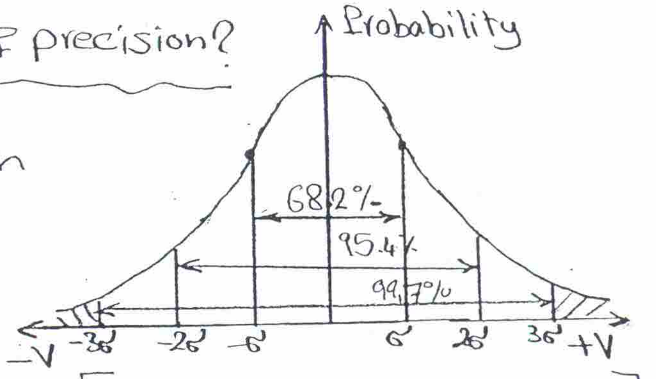
b) Probable Error (P) $= V_{\frac{n+1}{2}}$ if n is odd or $\frac{1}{2}\left(V_{\frac{n}{2}} + V_{\frac{n}{2}+1}\right)$ if n is even

* Sort errors ascending or descending without signs

c) Standard Deviation (σ) $\Longrightarrow$ Best measure

Standard Deviation

Why SD is the best measure of precision?



1- Has Geometrical interpretation
(2 inflection points at $\pm \sigma$)

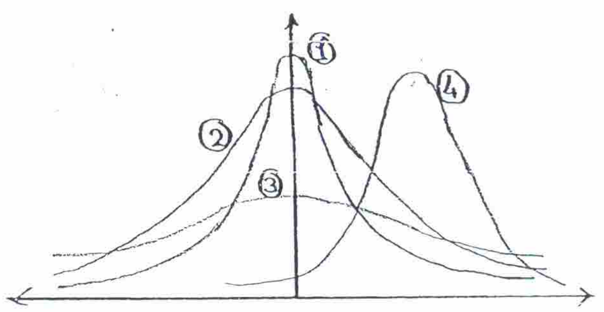
2- Has physical interpretation through area under the curve

Errors $> \pm 3\sigma$ are considered blunders (mistakes) and should be eliminated

- Errors between $-\sigma \rightarrow +\sigma = 68.2\%$
  Good observation
- Errors between $-2\sigma \rightarrow +2\sigma = 95.4\%$
  Bad observation
- Errors between $-3\sigma \rightarrow +3\sigma = 99.7\%$
  Very bad observation

3- To calculate it, the errors are squared, so large errors are magnified (spotted) and small errors are minimized

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{n} V_i^2}{n-1}}$$



Example: 1- very accurate
2- Less accurate
3- probability of small errors close to prob. of Large errors
4- occurrence of systematic error (bias)

Standard deviation

$$\sigma_x = \sqrt{\frac{\Sigma(V^2)}{n-1}}$$ S.D of single observation

It expresses the accuracy of single observation
of instrument
of observer

$$\sigma_{\bar{x}} = \sqrt{\frac{\Sigma(V^2)}{n(n-1)}}$$ S.D of the Mean or Standard error

$$\boxed{\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}}$$

σ_x^2 : variance of single obs.

$\sigma_{\bar{x}}^2$: Variance of the mean

$$\boxed{\sigma_{\bar{x}}^2 = \frac{\sigma_x^2}{n}}$$

$\sigma_{\bar{x}}$ (Less Value) $\Longrightarrow$ More Precise

To increase Precision:

1- Increase n (Measure Several times)

2- Use More accurate instruments

3- Utilize experienced observers.

Ex: A distance was measured and found to be : 18.26 – 18.27 – 18.24 – 18.23 – 18.25m. Find out the most probable value and a measure of error in it.

Solution.

	X_m	V_{cm}	$V^2_{cm^2}$
	18.26	1	1
	18.27	2	4
	18.24	-1	1
	18.23	-2	4
	18.25	0	0
Σ	91.25	0	10

$$\bar{X} = \frac{\Sigma X}{n} = \frac{91.25}{5} = 18.25\ m$$

$$\sigma_X = \pm\sqrt{\frac{10}{4}} = \pm 1.58\ cm$$

$$\sigma_{\bar{X}} = \pm\sqrt{\frac{10}{5 * 4}} = \pm 0.7\ cm$$

$\therefore$ M.P.V = 18.25 m $\pm$ 0.7 cm

* Relative error

للمقارنه بين أرصاد مختلفه لمعرفة أيهم أكثر دقه

$$R.E._{\bar{X}} = \frac{\sigma_{\bar{X}}}{\bar{X}} \quad \text{Unitless}$$

Ex: $M.P.V_X = 200\ Km \pm 4\ m$

$M.P.V_Y = 40m \pm 1mm$

Who is more precise X or Y ?

Sol:

$$R.E_X = \frac{4}{200 * 10^3} = \frac{1}{50000}$$

$$R.E_Y = \frac{1}{40 * 10^3} = \frac{1}{40000}$$

$\because R.E_X < R.E_Y \implies$ X is more Precise than Y

$$R.E_X = \frac{\sigma_X}{\bar{X}}$$

* The Weight "P" and the weighted mean "$\bar{X}$" *

* هى لنفس الكميه "X" طول أو زاويه ... وتم قياسها عدة مرات عن طريق أشخاص مختلفه أو شخص واحد فى ظروف مختلفه .

⊙ والمطلوب إيجاد المتوسط لجميع هذه الأرصاد سويا" .

Weight "P" $\propto 1/\sigma_{\bar{x}}^2$

$$P_1 : P_2 : \ldots : P_n$$

$$1/\sigma_{\bar{x}_1}^2 : 1/\sigma_{\bar{x}_2}^2 : \quad : 1/\sigma_{\bar{x}_n}^2$$

$$\therefore \bar{X} = \frac{P_1 * \bar{X}_1 + P_2 * \bar{X}_2 + \ldots P_n * \bar{X}_n}{P_1 + P_2 + \ldots P_n} = \frac{\Sigma(P_i \cdot \bar{X}_i)}{\Sigma P_i}$$

$$\sigma_x = \sqrt{\frac{\Sigma(P_i V_i^2)}{(n-1)}} \qquad \sigma_{\bar{x}} = \sqrt{\frac{\Sigma(P_i V_i^2)}{\Sigma P_i * (n-1)}}$$

check : $\Sigma P_i V_i = 0$

Ex: If the S.D of single angular obs. is $\pm 30''$, How many times must an angle be measured so its S.D doesn't exceed $\pm 9''$?

Sol: $n = \frac{\sigma_x^2}{\sigma_{\bar{x}}^2} = \frac{(30)^2}{(9)^2} = 11.111 \Rightarrow 12$ times

Ex: If the R.E of the mean of a measured distance is 1/7000. What would be the S.D of this distance in (Cm) if the distance equals to 580.54 m?

Sol: $R.E_{\bar{X}m} = \frac{\sigma_{\bar{x}}}{\bar{X}}$ $\quad \frac{1}{7000} = \frac{\sigma_{\bar{x}}}{580.54 * 100} \Rightarrow \sigma_{\bar{x}} = 8.3$ cm

Ex: The accuracy of a distance measuring instrument (EDM) for a single observation can be expressed as a R.E = (7mm + 30 ppm). This instrument has been used to measure a distance = 2.5 Km. How many times should we measure this distance to get its standard deviation to be less than ± 1.5 cm?

Sol: $\sigma_x = 7mm + \frac{30}{10^6} * 2.5 * 10^6 = 82$ mm

$n = \frac{\sigma_x^2}{\sigma_{\bar{x}}^2} = \frac{(8.2)^2}{(1.5)^2} = 29.88 = 30$ times

Ex: A distance of 500m measured with R.E = 20ppm, find S.D of mean?

Sol: $\sigma_{\bar{x}} = R.E * \bar{X} = \frac{20}{10^6} * 500 * 1000 = 10$ mm

Ex: Two persons measured the same distance and the observations were as follow:

P_1 : 45.19 - 45.08 - 45.23 - 45.05 - 45.11 - 45.22 - 45.10 - 45.11 - 45.24 - 45.17 m

P_2 : 45.27 - 45.28 - 45.20 - 45.18 - 45.24 - 45.12 - 45.22 - 45.23 - 45.13 - 45.18 m

* Compare between the two persons
* Find out the most probable value of that distance.

Solution:

P_1 $n = 10$

	X_m	V_{cm}	$V^2_{cm^2}$
	45.19	4	16
	45.08	-7	49
	45.23	8	64
	45.05	-10	100
	45.11	-4	16
	45.22	7	49
	45.10	-5	25
	45.11	-4	16
	45.24	9	81
	45.17	2	4
Σ	451.5	0	420

$\bar{X}_1 = (451.5)/10 = 45.15\,m$

P_2 $n = 10$

	X_m	V_{cm}	$V^2_{cm^2}$
	45.27	6.5	42.25
	45.28	7.5	56.25
	45.20	-0.5	0.25
	45.18	-2.5	6.25
	45.24	3.5	12.25
	45.12	-8.5	72.25
	45.22	1.5	2.25
	45.23	2.5	6.25
	45.13	-7.5	56.25
	45.18	-2.5	6.25
Σ	452.05	0	260.5

$\bar{X}_2 = 452.05/10 = 45.205\,m$

For P_1:

$\bar{X}_1 = 45.15\ m$

$\sigma_{x_1} = \pm\sqrt{\frac{420}{9}} = \pm 6.83\ cm$

$\sigma_{\bar{x}_1} = \pm\sqrt{\frac{420}{10 * 9}} = \pm 2.16\ cm$

$\therefore$ M.P.V for P_1 = $45.15\ m \pm 2.16\ cm$

For P_2:

$\bar{X}_2 = 45.205\ m$

$\sigma_{x_2} = \pm\sqrt{\frac{260.5}{9}} = 5.38\ cm$

$\sigma_{\bar{x}_2} = \pm\sqrt{\frac{260.5}{10 * 9}} = \pm 1.70\ cm$

$\therefore$ M.P.V for P_2 = $45.205\ m \pm 1.70\ cm$

$\because \sigma_{\bar{x}_1}\ (\pm 2.16\ cm) > \sigma_{\bar{x}_2}\ (\pm 1.70\ cm)$

$\therefore$ P_2 is more precise than P_1.

* To get the most probable value of that distance

$P_1 : P_2 = 1/(2.16)^2 : 1/(1.70)^2 = 1 : 1.6$

	X_m	V_{cm}	$P * V_{cm}$	$P * V^2_{cm^2}$
	45.15	-3.4	-3.4	11.56
	45.205	2.1	3.4	7.056
			0	18.616

$$\bar{X} = \frac{1 * 45.15 + 1.6 * 45.205}{1 + 1.6}$$

$$= 45.184 \text{ m}$$

$$\therefore \sigma_x = \pm\sqrt{\frac{18.616}{(2-1)}} = \pm 4.31 \text{ cm}$$

$$\sigma_{\bar{x}} = \pm\sqrt{\frac{18.616}{2.6\,(2-1)}} = \pm 2.68 \text{ cm}$$

$\therefore$ M.P.V $= 45.184 \text{ m} \pm 2.68 \text{ cm}$

Ex: Three persons measured the same distance and the observations were as Follow:

First P : 67.22 67.21 67.24 67.23 m

Second P : 67.20 67.18 67.24 67.22 67.19 67.23 m

Third P : 67.13 67.14 67.12 67.10 67.13 m

Compare between these persons. Then find out the most probable value of that distance.

Solution:

First P

	X_m	V_{cm}	$V^2_{cm^2}$
	67.22	-0.5	0.25
	67.21	-1.5	2.25
	67.24	1.5	2.25
	67.23	0.5	0.25
Σ	268.9	0	5

$$\bar{X}_1 = \frac{268.9}{4} = 67.225 \, m$$

$$\sigma_{X_1} = \pm \sqrt{\frac{5}{(4-1)}} = 1.29 \, cm$$

$$\sigma_{\bar{X}_1} = \pm \sqrt{\frac{5}{4 \times 3}} = 0.65 \, cm$$

$$\therefore M.P.V_1 = 67.225 \, m \pm 0.65 \, cm$$

Second P

X_m	V_{cm}	$V^2_{cm^2}$
67.20	-1	1
67.18	-3	9
67.24	3	9
67.22	1	1
67.19	-2	4
67.23	2	4
Σ 403.26	0	28

$$\bar{X}_2 = \frac{403.26}{6} = 67.21\ m$$

$$\sigma_{x_2} = \pm\sqrt{\frac{28}{(6-1)}} = \pm 2.37\ cm$$

$$\sigma_{\bar{x}_2} = \pm\sqrt{\frac{28}{6 \times 5}} = \pm 0.97\ cm$$

$$\therefore M.P.V_2 = 67.21\,m \pm 0.97\ cm$$

X_m	V_{cm}	$V^2_{cm^2}$
67.13	0.6	0.36
67.14	1.6	2.56
67.12	-0.4	0.16
67.10	-2.4	5.76
67.13	0.6	0.36
Σ 335.62	0	9.2

$$\bar{X}_3 = \frac{335.62}{5} = 67.124\ m$$

$$\sigma_{x_3} = \pm\sqrt{\frac{9.2}{(5-1)}} = \pm 1.52\ cm$$

$$\sigma_{\bar{x}_3} = \pm\sqrt{\frac{9.2}{5 \times 4}} = \pm 0.68\ cm$$

$$\therefore M.P.V_3 = 67.124\ m \pm 0.68\,cm$$

$$\because \sigma_{\bar{x}_1} < \sigma_{\bar{x}_3} < \sigma_{\bar{x}_2}$$

$\therefore$ First P > Third P > Second P (Precision)

* To get the most probable value of that distance

$$P_1 : P_2 : P_3 = 1/(0.65)^2 : 1/(0.97)^2 : 1/(0.68)^2$$

$$= 2.23 : 1 : 2.03$$

	X_m	V_{cm}	$P \times V_{cm}$	$P \times V^2_{cm^2}$
	67.225	4.2	9.366	39.34
	67.210	2.7	2.7	7.29
	67.124	-5.9	-11.977	70.66
Σ	201.559		0	117.29

$$\bar{X} = \frac{67.225 \times 2.23 + 67.210 \times 1 + 67.124 \times 2.03}{(2.23 + 1 + 2.03)}$$

$$\therefore \bar{X} = 67.183 \text{ m}$$

$$\therefore \sigma_x = \pm \sqrt{\frac{117.29}{(3-1)}} = \pm 7.66 \text{ cm}$$

$$\sigma_{\bar{x}} = \pm \sqrt{\frac{117.29}{5.26 \times 2}} = \pm 3.34 \text{ cm}$$

$$\therefore M.P.V = 67.183 \text{ m} \pm 3.34 \text{ cm}$$

EX: The same length of line AB was measured by 4 observing groups and the results as follow :

$X_1 = 670.30$ m $\qquad \sigma_{x_1}^2 = 0.5\ cm^2$

$X_2 = 670.34$ m $\qquad \sigma_{x_2}^2 = 0.25\ cm^2$

$X_3 = 670.38$ m $\qquad \sigma_{x_3}^2 = 1.00\ cm^2$

$X_4 = 670.36$ m $\qquad \sigma_{x_4}^2 = 0.5\ cm^2$

* Compute the weighted mean of the distance AB & its S.D.

Solution :

$$P_1 : P_2 : P_3 : P_4$$

$$1/0.5 : 1/0.25 : 1/1 : 1/0.5$$

$$2 : 4 : 1 : 2$$

670. X_i Observation	V_i $X_i - 0.34$	$P_i * V_i$	$P_i \cdot V_i^2$
0.30	−0.04	−0.08	$3.2 * 10^{-3}$
0.34	0	0	0
0.38	0.04	0.04	$1.6 * 10^{-3}$
0.36	0.02	0.04	$0.8 * 10^{-3}$
	Σ	Zero	$5.6 * 10^{-3}$

$$\bar{X}_m = \frac{\Sigma W_i \cdot X_i}{\Sigma W_i} = \frac{(0.30 * 2) + (0.34 * 4) + (0.38 * 1) + (0.36 * 2)}{(2 + 4 + 1 + 2)} = 0.34\ m$$

$$\bar{X}_m = 670.34 m \quad , \quad \sigma_{\bar{X}_m} = \sqrt{\frac{\Sigma P_i V_i^2}{(n-1)\Sigma P_i}} = \sqrt{\frac{5.6 * 10^{-3}}{3 * 9}} = \pm 0.0144\ m$$

Ex: An angle θ was measured by two observers using the same Theodolite and the results as follow:

	1	2	3	4	5	6	7	8	9	10
obs.1	55° 30' 19"	8"	23"	5	11	22	10	11	24	17"
Obs.2	55° 13' 17"	26"	20"	18	40	19	22	20	10	8"

a- Compute the mean, variance, S.Deviation of each sample.

b- Compute the variance & S.deviation of the mean.

c- Which set is more precise ? why ?

d- Use both Observers to compute the weighted mean and its standard deviation.

Solution:

L_1	V_1	V_1^2	L_2	V_2	V_2^2
19	4	16	17	-3	9
8	-7	49	26	6	36
23	8	64	20	0	0
5	-10	100	18	-2	4
11	-4	64	40	20	400
22	7	49	19	-1	1
10	-5	25	22	2	4
11	-4	16	20	0	0
24	9	81	10	-10	100
17	2	4	8	-12	144
Σ 150	0	420	200	0	698

$$\bar{L}_1 = \frac{\Sigma L_1}{n} = \frac{150}{10} = 15'' \quad (55° \, 30' \, 15'')$$

$$\sigma_1^2 = \frac{\Sigma V_1^2}{n-1} = \frac{420}{9} = 46.67 \text{ sec}^2$$

$$\sigma_1 = \sqrt{\sigma_1^2} = 6.83 \text{ sec}$$

$$\hat{\sigma}_1^2 = \frac{\Sigma V_1^2}{n(n-1)} = \frac{420}{10 * 9} = 4.67 \text{ sec}^2$$

$$\hat{\sigma}_1 = \sqrt{\hat{\sigma}_1^2} = 2.16 \text{ sec}$$

Same for L_2:

$$\bar{L}_2 = 55° \, 13' \, 20''$$

$\sigma_2^2 = 77.56 \text{ sec}^2$ $\quad \sigma_2 = 8.8$ sec

$\hat{\sigma}_2^2 = 7.76 \text{ sec}^2$ $\quad \hat{\sigma}_2 = 2.78$ sec

c) $\hat{\sigma}_1 = 2.16$ sec $< \hat{\sigma}_2 = 2.78$ sec

$\therefore$ Observer 1 is more precise.

d) $P_1 : P_2 = 1/\hat{\sigma}_1^2 : 1/\hat{\sigma}_2^2 = \frac{1}{(2.16)^2} : \frac{1}{(2.78)^2} = 1.66 : 1$

$\bar{L}_1 = 55° \, 30' \, 15''$ $\qquad \bar{L}_2 = 55° \, 13' \, 20''$

$$\hat{X}_m = \frac{1.66 * (30' \, 15'') + 1 * (13' \, 20'')}{1.66 + 1} = 23' \, 53.4''$$

$\therefore \bar{X}_m = 55° \, 23' \, 53.4''$

$\therefore V_1 = 30' \, 15'' - 23' \, 53.4'' = 6' \, 21.6''$

$V_2 = 13' \, 20'' - 23' \, 53.4'' = -10' \, 33.4''$

$\Sigma P_i V_i = 1.66 * (6' \, 21.6'') + 1 * (-10' \, 33.4'') = 0$

$$\sigma_{\bar{X}m} = \sqrt{\frac{\Sigma P_i V_i^2}{(n-1) \, \Sigma P_i}} = \sqrt{\frac{1.66 \times (6' \, 21.6'')^2 + 1 * (-10' \, 33.4'')^2}{(2-1) * (1.66+1)}} = 8' \, 11.6''$$

$$\boxed{\therefore \bar{X}_m = 55° \, 23' \, 53.4'' \pm 8' \, 11.6''}$$

* Propagation of errors *

* One unknown is function of more than one observation

$$X = F(L)$$

⇒ Variance law of error propagation

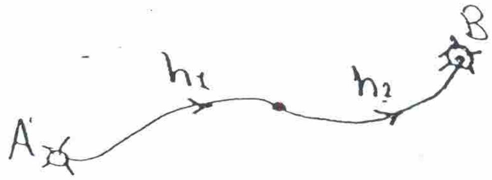
$$\sigma_{\hat{x}}^2 = \left(\frac{\partial f}{\partial L_1}\right)^2 \sigma_{L_1}^2 + \left(\frac{\partial f}{\partial L_2}\right)^2 \sigma_{L_2}^2 + \text{-------------}$$

Ex: The attaced figure shows a leveling line runs from the Bench mark A of a fixed elevation $H_A = 40.00$ m to point (B) whose Elevation is required. given that:

$h_1 = 3.15$ m $\pm$ 3 cm , $h_2 = 5.85$ m $\pm$ 5 cm

* Calculate H_B & its standard deviation.

Solution



$$H_B = H_A + h_1 + h_2$$

$$= 40 + 3.15 + 5.85 = 49 \text{ m}$$

$$\sigma_{\hat{H}_B}^2 = \left(\frac{\partial H_B}{\partial h_1}\right)^2 \sigma_{\hat{h}_1}^2 + \left(\frac{\partial H_B}{\partial h_2}\right)^2 \sigma_{\hat{h}_2}^2$$

$$\frac{\partial H_B}{\partial h_1} = 1 \qquad \frac{\partial H_B}{\partial h_2} = 1$$

$$\therefore \sigma_{\hat{H}_B}^2 = (1)^2(3)^2 + (1)^2(5)^2 = 34 \text{ cm}^2 \Rightarrow \sigma_{\hat{H}_B} = 5.83 \text{ cm}$$

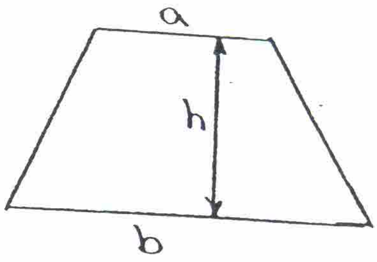
$$\boxed{\therefore H_B = 49 \text{ m} \pm 5.83 \text{ cm}}$$

Ex: A piece of land has a trapezoidal shape with two parallel sides a, b and the height of trapezoid is h. The results of measurments are :

$a = 84.22\,m \pm 0.12\,m$

$b = 150.08\,m \pm 0.14\,m$

$h = 40.00\,m \pm 0.10\,m$



* Compute the area & its standard deviation.

Solution : $A = \left(\frac{a+b}{2}\right)h = \left(\frac{84.22 + 150.08}{2}\right) * 40 = 4{,}686\ m^2$

$$\sigma_{\hat{A}}^2 = \left(\frac{\partial A}{\partial a}\right)^2 \sigma_{\hat{a}}^2 + \left(\frac{\partial A}{\partial b}\right)^2 \sigma_{\hat{b}}^2 + \left(\frac{\partial A}{\partial h}\right)^2 \sigma_{\hat{h}}^2$$

$$\frac{\partial A}{\partial a} = \frac{h}{2} = 20\ m \qquad \frac{\partial A}{\partial b} = \frac{h}{2} = 20\,m \qquad \frac{\partial A}{\partial h} = \frac{a+b}{2} = 117.15\,m$$

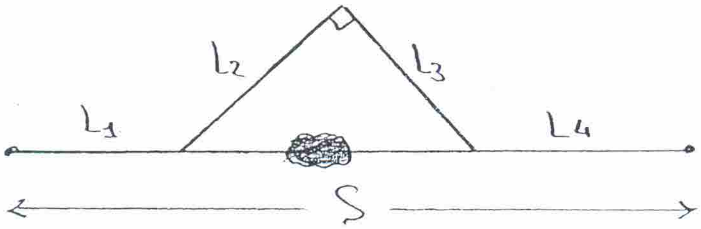
$$\therefore \sigma_{\hat{A}}^2 = (20)^2 (0.12)^2 + (20)^2 (0.14)^2 + (117.15)^2 (0.1)^2 = 150.84\ m^4$$

$$\therefore \sigma_{\hat{A}} = \sqrt{150.84} = 12.28\ m^2$$

$$\boxed{\therefore A = 4{,}686 \pm 12.28\ m^2}$$

Ex: $\bar{L}_1 = 305.25\,m \pm 0.4\,m^2$ $\bar{L}_2 = 60\,m \pm 0.2\,m^2$ $\bar{L}_3 = 80\,m \pm 0.2\,m^2$

$\bar{L}_4 = 249.75\,m \pm 0.4\,m^2$ Find the length S with its S.D.



Sol:

$$S = L_1 + (L_2^2 + L_3^2)^{1/2} + L_4 = 655\,m$$

$$\frac{\partial S}{\partial L_1} = 1 \text{ U.L} \qquad \frac{\partial S}{\partial L_2} = \frac{1}{2}(L_2^2 + L_3^2)^{-1/2}(2L_2) = \frac{L_2}{(L_2^2 + L_3^2)^{1/2}} = 0.6 \text{ U.L}$$

$$\frac{\partial S}{\partial L_3} = \frac{L_3}{(L_2^2 + L_3^2)^{1/2}} = 0.8 \text{ U.L} \qquad \frac{\partial S}{\partial L_4} = 1 \text{ U.L}$$

$$\sigma_S^2 = (1)^2(0.4) + (0.6)^2(0.2) + (0.8)^2(0.2) + (1)^2(0.4) = 1\,m^2$$

$$\sigma_S = 1\,m$$

$$\therefore S = 655\,m \pm 1\,m$$

C.S.C
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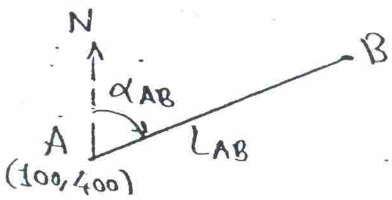
Ex: The coordinates of point A are (100m, 400m). The distance from point A to another point B (L_{AB}) was measured as well as its azimuth (α_{AB}) and the results are:

$L_{AB} = 150.60 \text{ m} \pm 10 \text{ cm}$ $\quad\quad$ $\alpha_{AB} = 30° 30' 00'' \pm 4''$

(Both observations are not correlated).

* Compute the easting of point B (E_B) and its S.D.

Solution



$$E_B = E_A + L_{AB} \sin \alpha_{AB}$$

$$= 100 + 150.6 * \sin(30° 30' 00'') = 176.435 \text{ m}$$

$$\hat{\sigma}_{E_B}^2 = \left(\frac{\partial E_B}{\partial L_{AB}}\right)^2 \hat{\sigma}_{L_{AB}}^2 + \left(\frac{\partial E_B}{\partial \alpha_{AB}}\right)^2 \hat{\sigma}_{\alpha_{AB}}^2$$

$$\frac{\partial E_B}{\partial L_{AB}} = \sin \alpha_{AB} = 0.51 \text{ (Unitless)}$$

$$\frac{\partial E_B}{\partial \alpha_{AB}} = L_{AB} \cdot \cos \alpha_{AB} = 129.76 \text{ m}$$

$$\therefore \hat{\sigma}_{E_B}^2 = \underset{U.L}{(0.51)^2} * \underset{cm^2}{(10)^2} + \underset{cm^2}{(129.76 * 100)^2} * \underset{U.L}{\left(\frac{4}{206265}\right)^2} = 26.07 \text{ cm}^2$$

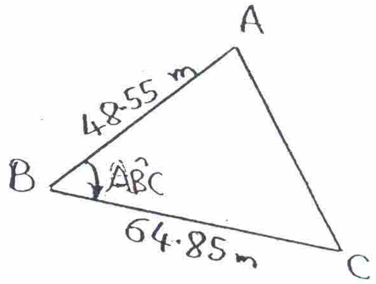
$$\because \rho = 206265 = \frac{180}{\pi} * 60 * 60$$

$$\boxed{E_B = 176.435 \text{ m} \pm 5.1 \text{ cm}}$$

Ex: In the triangle ABC, the following were measured as:

$AB = 48.55\ m \pm 3cm$ $BC = 64.85 \pm 4cm$ $ABC = 40^\circ 30' 45'' \pm 6''$

Compute the distance AC and its S.D.



Sol:

$$AC = (AB^2 + BC^2 - 2\ AB\ BC \cos ABC)^{1/2}$$

$$= 42.134\ m$$

$$\frac{\partial AC}{\partial AB} = \frac{1}{2}(AB^2 + BC^2 - 2AB\ BC \cos ABC)^{-1/2}(2AB - 2BC \cos ABC)$$

$$= \frac{AB - BC \cos ABC}{AC} = -0.018\ U.L$$

$$\frac{\partial AC}{\partial BC} = \frac{BC - AB \cos ABC}{AC} = 0.663\ U.L$$

$$\frac{\partial AC}{\partial ABC} = \frac{AB\ BC \sin ABC}{AC} = 48.542\ m$$

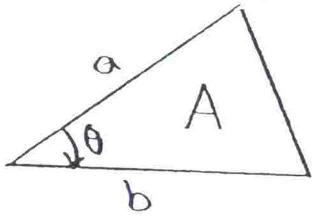
$$\sigma_{AC}^2 = \left(\frac{\partial AC}{\partial AB}\right)^2 \sigma_{AB}^2 + \left(\frac{\partial AC}{\partial BC}\right)^2 \sigma_{BC}^2 + \left(\frac{\partial AC}{\partial ABC}\right)^2 \sigma_{ABC}^2$$

$$= (-0.018)^2 (3)^2 + (0.663)^2 (4)^2 + \left(\frac{48.542 * 100}{206265}\right)^2 (6)^2$$

$$= 7.056\ cm^2 \implies \sigma_{AC} = 2.656\ cm$$

$$\therefore AC = 42.134 \pm 2.656\ cm$$

Ex: $a = 120.80\ m \pm 4\ cm$



$b = 180.80\ m \pm 6\ cm$

$\theta = 60^\circ 30' 45'' \pm 15''$

* Required Area & its Standard deviation.

Solution: $A = \frac{1}{2} . a . b . \sin\theta = \frac{1}{2} * 120.8 * 180.80 * \sin(60^\circ 30' 45'')$

$\therefore A = 9,505.736\ m^2$

$$\sigma_{\hat{A}}^2 = \left(\frac{\partial A}{\partial a}\right)^2 \sigma_{\hat{a}}^2 + \left(\frac{\partial A}{\partial b}\right)^2 \sigma_{\hat{b}}^2 + \left(\frac{\partial A}{\partial \theta}\right)^2 \sigma_{\hat{\theta}}^2$$

$$\frac{\partial A}{\partial a} = \frac{1}{2} . b . \sin\theta = 78.69\ m$$

$$\frac{\partial A}{\partial b} = \frac{1}{2} . a . \sin\theta = 52.576\ m$$

$$\frac{\partial A}{\partial \theta} = \frac{1}{2} . a . b . \cos\theta = 5,375.349\ m^2$$

$$\therefore \sigma_{\hat{A}}^2 = \underbrace{(78.69)^2}_{m^2} \underbrace{(0.04)^2}_{m^2} + \underbrace{(52.576)^2}_{m^2} \underbrace{(0.06)^2}_{m^2} + \underbrace{(5375.349)^2}_{m^4} \underbrace{\left(\frac{15}{206265}\right)^2}_{U.L}$$

$\sigma_{\hat{A}}^2 = 20.011\ m^4 \implies \sigma_{\hat{A}} = 4.473\ m^2$

$$\boxed{\therefore A = 9,505.736\ m^2 \pm 4.473\ m^2}$$

* For the Previous example, due to high price of that Land, it is required to improve the precision of that area to achieve a precision not less than 4 m^2 If you know that you have 30" theodolite and an EDM instrument (5mm + 20ppm). Suggest an observing technique to achieve that precision.

Solution:

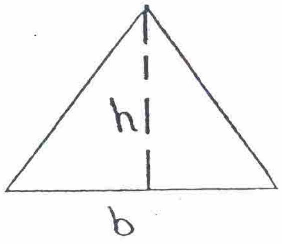
$$\sigma_{\hat{A}}^2 = \overset{a}{(78.69)^2 (0.04)^2} + \overset{b}{(52.576)^2 (0.06)^2} + \overset{\theta}{(5375.349)^2 \left(\frac{15}{206265}\right)^2}$$

$$= \underset{\sim}{9.91} + \underset{\sim}{9.95} + \underset{\sim}{0.17}$$

∵ term of b is the most Factor affecting the precision of the area, therefore we can increase the no. of observations for b or we use better EDM to measure this distance.

Important Examples :

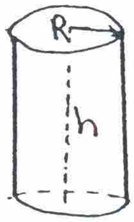
1) $A = \frac{1}{2} \cdot b \cdot h$



$$\sigma_A^2 = \left(\frac{\partial A}{\partial h}\right)^2 (\sigma_h)^2 + \left(\frac{\partial A}{\partial b}\right)^2 \sigma_b^2$$

$\frac{\partial A}{\partial b} = \frac{1}{2} * h$ --- m $\frac{\partial A}{\partial h} = 1/2 * b$ --- m

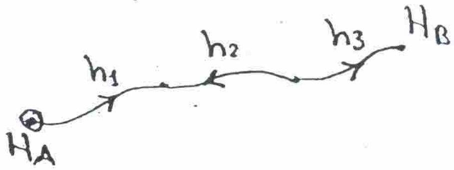
2) $V = \pi R^2 \cdot h$



$$\sigma_V^2 = \left(\frac{\partial V}{\partial R}\right)^2 \sigma_R^2 + \left(\frac{\partial V}{\partial h}\right)^2 \sigma_h^2$$

$\frac{\partial V}{\partial R} = 2\pi . R . h$ ---- m^2 $\frac{\partial V}{\partial h} = \pi R^2$ ---- m^2

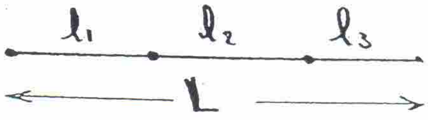
3) $H_B = H_A + h_1 - h_2 + h_3$



$$\sigma_{H_B}^2 = \left(\frac{\partial H_B}{\partial h_1}\right)^2 \sigma_{h_1}^2 + \left(\frac{\partial H_B}{\partial h_2}\right)^2 \sigma_{h_2}^2 + \left(\frac{\partial H_B}{\partial h_3}\right)^2 \sigma_{h_3}^2$$

$\frac{\partial H_B}{\partial h_1} = 1$ (U.L) $\frac{\partial H_B}{\partial h_2} = -1$ (U.L) $\frac{\partial H_B}{\partial h_3} = 1$ (U.L)

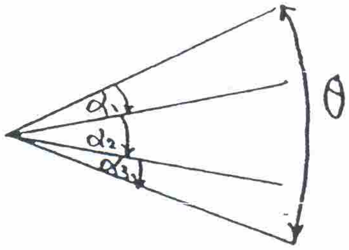
4) $L = l_1 + l_2 + l_3$



$$\sigma_L^2 = \left(\frac{\partial L}{\partial l_1}\right)^2 \sigma_{l_1}^2 + \left(\frac{\partial L}{\partial l_2}\right)^2 \sigma_{l_2}^2 + \left(\frac{\partial L}{\partial l_3}\right)^2 \sigma_{l_3}^2$$

$\frac{\partial L}{\partial l_1} = \frac{\partial L}{\partial l_2} = \frac{\partial L}{\partial l_3} = 1$ (U.L)

5) $\theta = \alpha_1 + \alpha_2 + \alpha_3$



$$\sigma_\theta^2 = \left(\frac{\partial \theta}{\partial \alpha_1}\right)^2 \sigma_{\alpha_1}^2 + \left(\frac{\partial \theta}{\partial \alpha_2}\right)^2 \sigma_{\alpha_3}^2 + \left(\frac{\partial \theta}{\partial \alpha_3}\right)^2 \sigma_{\alpha_3}^2$$

$\frac{\partial \theta}{\partial \alpha_1} = \frac{\partial \theta}{\partial \alpha_2} = \frac{\partial \theta}{\partial \alpha_3} = 1$ (U.L)

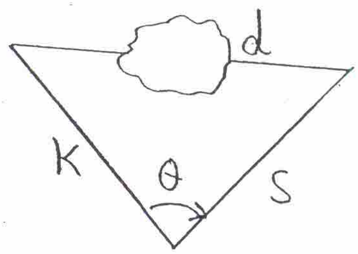
Preanalysis

* Process done in the office before going to the field to get the no. of observations required for each observation quantity in order to achieve desired accuracy for an unknown.

This is done using the variance law:

$$\sigma_X^2 = \left(\frac{\partial X}{\partial L_1}\right)^2 \sigma_{L_1}^2 + \left(\frac{\partial X}{\partial L_2}\right)^2 \sigma_{L_2}^2 + \cdots\cdots + \left(\frac{\partial X}{\partial L_n}\right)^2 \sigma_{L_n}^2$$

Ex: It is required to determine the distance "d" so that, its S.D should not exceed 0.5 cm. Given that: K = 115 m, S = 136 m, θ = 50°. Note that S.D of EDM = 1.5 cm and Theodolite = 10".



Sol:

$$d = (K^2 + S^2 - 2KS\cos\theta)^{1/2} \simeq 107.77\ m$$

$$\frac{\partial d}{\partial K} = \frac{K - S\cos\theta}{d} = 0.256\ U.L$$

$$\frac{\partial d}{\partial S} = \frac{S - K\cos\theta}{d} = 0.576\ U.L$$

$$\frac{\partial d}{\partial \theta} = \frac{KS\sin\theta}{d} = 111.171\ m$$

$$\therefore (0.5)^2 = (0.256)^2(\sigma_K)^2 + (0.576)^2(\sigma_S)^2 + \left(\frac{111.17 * 100}{206265}\right)^2 \sigma_\theta^2$$

First Trial: $K:S:\theta = 1:1:1$

Assume $(0.5)^2$ is equally distributed on observations

For K: $\frac{(0.5)^2}{3} = (0.256)^2 * \sigma_K^2 \Rightarrow \sigma_K^2 = 1.271\ cm^2 \Rightarrow n_K = \frac{1.5^2}{1.271} = 2$

For S: $\frac{(0.5)^2}{3} = (0.576)^2 * \sigma_S^2 \Rightarrow \sigma_S^2 = 0.251\ cm^2 \Rightarrow n_S = \frac{1.5^2}{0.251} = 9$

For θ: $\frac{(0.5)^2}{3} = \left(\frac{111.17 * 100}{206265}\right)^2 \sigma_\theta^2 \Rightarrow \sigma_\theta^2 = 28.68\ sec^2 \Rightarrow n_\theta = \frac{10^2}{28.68} = 4$

Second Trial: $K:S:\theta = 2:9:4 = \frac{2}{15} : \frac{9}{15} : \frac{4}{15}$

For K: $(0.5)^2 * \frac{2}{15} = (0.256)^2 * \sigma_K^2 \Rightarrow \sigma_K^2 = 0.508\ cm^2 \Rightarrow n_K = \frac{1.5^2}{0.508} = 5$

For S: $(0.5)^2 * \frac{9}{15} = (0.576)^2 * \sigma_S^2 \Rightarrow \sigma_S^2 = 0.452\ cm^2 \Rightarrow n_S = \frac{1.5^2}{0.452} = 5$

For θ: $(0.5)^2 * \frac{4}{15} = \left(\frac{111.17 * 100}{206265}\right)^2 \sigma_\theta^2 \Rightarrow \sigma_\theta^2 = 22.95\ sec^2 \Rightarrow n_\theta = \frac{10^2}{22.95} = 5$