EL-MOASSER

in

Statics

Final Examinations

THE BOOKLET

By a group of supervisors

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1. Statics

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Preface

Thanks to God who helped us to introduce one of our famous series "El Moasser" in mathematics.

We introduce this book to our colleagues.

We also introduce it to our students to help them study mathematics.

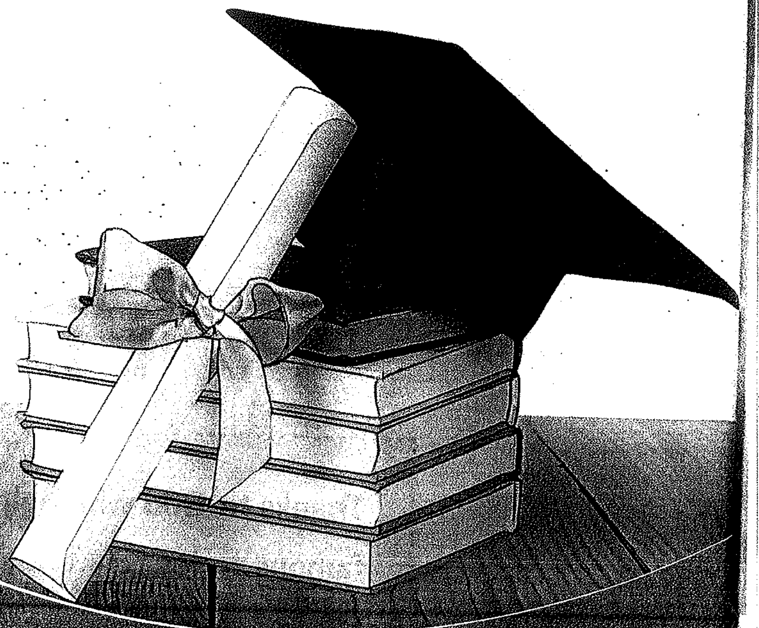
In fact, this book is the outcome of more than thirty years experience in the field of teaching mathematics.

This book will make students aware of all types of questions.

We would like to know your opinions about the book hoping that it will win your admiration.

We will be grateful if you send us your recommendations and your comments.

The Authors



Contents

Egypt Exams and Model Examinations

Each model examination contains 18 questions as the following :

- **10 multiple choices questions.**
- **8 essay questions.**

Remark : Among the 8 essay questions there are two questions each of them contains two requirements, student should answer only one of the two requirements.

تعليمات مهمة

- عدد أسئلة كراسة الامتحان (١٨) سؤالًا.
- عدد صفحات كراسة الامتحان (....) صفحة.
- تأكد من ترقيم الأسئلة ، ومن عدد صفحات كراسة الامتحان ، فهى مسئوليتك.
- زمن الاختبار (ساعتان).
- الدرجة الكلية للاختبار (٣٠) درجة.

عزيزى الطالب ... اقرأ هذه التعليمات بعناية :

١ اقرأ التعليمات جيدًا سواء فى مقدمة كراسة الامتحان أو مقدمة الأسئلة ، وفى ضوئها أجب عن الأسئلة.

٢ اقرأ السؤال بعناية ، وفكر فيه جيدًا قبل البدء فى إجابته.

٣ استخدم القلم الجاف الأزرق للإجابة ، والقلم الرصاص فى الرسومات ، وعدم استخدام مزيل الكتابة.

٤ عند إجابتك للأسئلة المقالية ، أجب فى المساحة المخصصة للإجابة وفى حالة الحاجة لمساحة أخرى يمكن استكمال الإجابة فى صفحات المسودة مع الإشارة إليها ، وإن إجابتك بأكثر من إجابة سوف يتم تقديرها.

مثال :

____

____

____

٥ عند إجابتك عن الأسئلة المقالية الاختيارية أجب عن **[a]** أو **[b]** فقط.

٦ عند إجابتك عن أسئلة الاختيار من متعدد إن وجدت :

ظلل الدائرة ذات الرمز الدال على الإجابة الصحيحة تظليلًا كاملًا لكل سؤال.

مثال : الإجابة الصحيحة (c) مثلًا.

(a)

(b)

(c) ● الإجابة الصحيحة مثلًا

(d)

- فى حالة ما إذا أجبت إجابة خطأ ، ثم قمت بالشطب وأجبت إجابة صحيحة تحسب الإجابة صحيحة.
- وفى حالة ما إذا أجبت إجابة صحيحة ، ثم قمت بالشطب وأجبت إجابة خطأ تحسب الإجابة خطأ.

ملحوظة :

فى حالة الأسئلة الموضوعية (الاختيار من متعدد) إذا تم التظليل على أكثر من رمز أو تم تكرار الإجابة ؛ تعتبر الإجابة خطأ.

٧ يسمح باستخدام الآلة الحاسبة.

٨ $g = 9.8\ m./sec^2 = 980\ cm./sec^2$

٩ $(\hat{i}\ ,\hat{j}\ ,\hat{k})$ are a right set of unit vectors.

Egypt exam 1st session 2017 on statics

Answer the following questions :

1 If $\overrightarrow{F_1} = 4\hat{i} + b\hat{j}$, $\overrightarrow{F_2} = a\hat{i} - 6\hat{j}$ are two forces form a couple , then a + b = ...

(a) – 10

(b) 2

(c) – 2

(d) 10

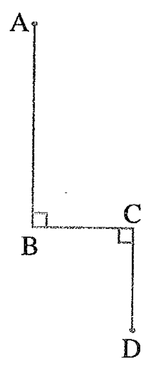
2 ABCD is a trapezium in which $\overline{AD} \mathbin{/\mkern-6mu/} \overline{BC}$, m ($\angle$ B) = 90° , AB = 12 cm. , BC = 18 cm. , AD = 9 cm. , force of magnitudes 20 , 60 , 50 , 120 and $30\sqrt{13}$ gm.wt. act along $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ and $\overrightarrow{AC}$ respectively. Prove that the system is equivalent to a couple and find its moment.

3. ABCD is a thin rectangular lamina in which AB = 18 cm. , BC = 24 cm. , its weight equals 20 newton acting at the point of intersection of the two diagonals. The lamina is suspended by a thin pin passing through a small hole near the vertex D so that its plane is vertical. If a couple the magnitude of its moment is 150 newton. cm. and its direction is perpendicular to the plane of the lamina acted on it. Find the inclination angle of $\overline{DB}$ to the vertical in the position of equilibrium.

4. The centre of gravity of the system consists of two masses 4 , 8 kg. and the distance between them is 6 m. lies at a distance .... m. from the first mass.

(a) 3
(b) 4
(c) 2
(d) 5

5. **In the opposite figure :**



If ABCD is a uniform wire of length 32 cm. where : AB = 2 BC = 2 CD = 16 cm. , then the distance from the centre of gravity of the wire to each of $\overleftrightarrow{BC}$ and $\overleftrightarrow{BA}$ respectively is ....

(a) (3 , 3)
(b) (4 , 4)
(c) (3 , 5)
(d) (4 , 8)

6 A fine lamina of uniform thickness and density , in the form of a trapezium ABCD in which m ($\angle$ A) = m ($\angle$ D) = 90° , CD = 40 cm. , AD = 60 cm. , AB = 120 cm. Find the distance from the centre of gravity of the lamina and each of $\overline{AD}$ and $\overline{AB}$

7 A body of weight 35 newton is placed on a rough horizontal plane , two horizontal forces act on the body of magnitudes 6 newton and 10 newton and including between them an angle of measure 60°

If the body is about to move , then the coefficient of static friction equals ....

(a) $\frac{2}{5}$

(b) $\frac{1}{14}$

(c) $\frac{3}{7}$

(d) $\frac{1}{10}$

8 If a body of weight 4 newton is placed on a rough horizontal plane , the coefficient of static friction between the plane and the body = $\frac{1}{4}$, and a horizontal force acts on the body trying to move it, then the static friction force $\in$ ....

(a) $[\frac{1}{4} , 4]$

(b) $[1 , \infty[$

(c) $]0 , 1]$

(d) $[0 , \frac{1}{4}]$

9 ***Answer one of the following items :***

[a] If the force $\overrightarrow{F} = 2\hat{i} + 3\hat{j} - \hat{k}$ acts at point A (1 , – 1 , 4) find the moment vector of the force $\overrightarrow{F}$ about point B (2 , – 3 , 1) , then calculate the length of the perpendicular drawn from the point B on the line of action of the force.

[b] The forces $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{F_2} = 5\hat{i} - 2\hat{j}$, $\overrightarrow{F_3} = -3\hat{i} + 2\hat{j}$ act at the point A (– 3 , 5) , find the moments vector of the resultant of these forces about the point B (1 , 7) and the distance between the point B and the line of action of the resultant.

10 If $\overrightarrow{F}$ = (2 , – 3 , 4) acts at the point (1 , 1 , 1) , then the component of the moment of $\overrightarrow{F}$ about y-axis equals ....

(a) 7

(b) – 2

(c) – 5

(d) 2

11 **In the opposite figure :**

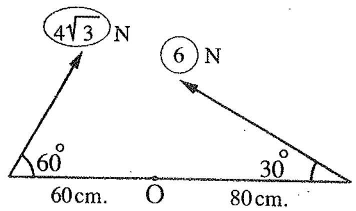
The sum of the moments of the forces about the point O equals .... N.cm.

(a) – 120

(b) 120

(c) 240

(d) – 360



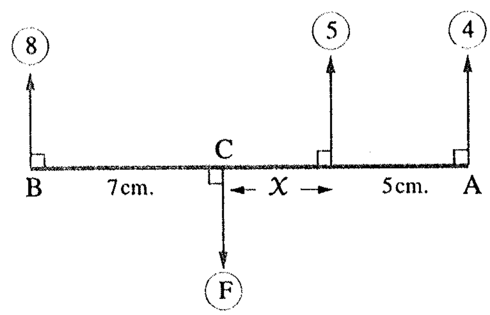
12 A body of weight 400 gm.wt. is placed on rough plane inclined to the horizontal by an angle of measure 30° , the coefficient of static friction between it and the body is $\frac{\sqrt{3}}{4}$ A force of magnitude 50 gm.wt. acts on it in the direction of the line of the greatest slope of the plane upwards. If the body is in equilibrium , then determine the friction force and show whether the body is about to move or not ?

13 $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two parallel forces act in opposite directions. If $F_1 = 7$ newton , $F_2 = 9$ newton , and the distance between the resultant and the second force equals 35 cm. , then the distance between the two forces equals .... cm.

(a) 10
(b) 16
(c) 35
(d) 70

14 **In the opposite figure :**

If $\overline{AB}$ is a rod is in equilibrium horizontally , then the distance $X =$ .... cm.

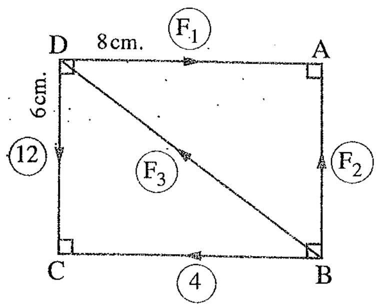


(a) 56
(b) 36
(c) 27
(d) 4

15 Find the magnitude and the point of action of the resultant of two parallel forces of magnitude 7 newton , 12 newton act at A and B in two opposite directions such that AB = 20 cm.

16 **In the opposite figure :**

If the magnitude of the forces in newton and the system is in equilibrium , then F_2 = ... newton.



(a) 16
(b) 5
(c) 3
(d) 8

17 $\overline{AB}$ is a uniform wooden board of mass 10 kg. and length 4 metres rests horizontally on two supports one of them at A and the other at a point distant 1 metre from B Show at which distance a 50 kg.wt. child can stand on the board in order to the reactions on the two supports get equal.

18 ***Answer one of the following items :***

[a] $\overline{AB}$ is a rod of negligible weight and of length 210 cm. , is hanged at A to a hinge fixed at a vertical wall. It carried at B a weight of magnitude 120 newton.

The rod is kept in a horizontal position by means of a light string attached at the end B of the rod , its other end is fixed at a point on the wall lying vertically above A. If the string inclined to the horizontal at an angle of measure 30° , find the magnitude of the tension in the string and the magnitude of the reaction of the hinge.

[b] A uniform ladder of weight 20 kg.wt. rests at one of its ends on a rough horizontal ground and with its other end against a smooth vertical wall such that the ladder equilibrium in a vertical plane , inclining to the horizontal at an angle of measure 60°

If the coefficient of friction between the ladder and the ground is $\frac{1}{2\sqrt{3}}$, prove that the maximum distance which a girl of weight 60 kg.wt. can ascend the ladder equals half the length of the ladder.

Egypt exam 2nd session 2017 on statics

Answer the following questions :

1 Two parallel forces of magnitudes 40 , 100 newton act in two opposite directions. If the distance between their lines of action equals 240 cm. , then find their resultant and its point of action.

2 If $\overrightarrow{F_1} = 3\,\overrightarrow{i} - b\,\overrightarrow{j}$, $\overrightarrow{F_2} = a\,\overrightarrow{i} - 5\,\overrightarrow{j}$ are two forces form a couple , then (a , b) = ...

(a) (3 , – 4)

(b) (3 , 5)

(c) (– 3 , 5)

(d) (– 3 , – 5)

3 ***Answer one of the following items :***

[a] A uniform rod $\overline{AB}$ of length 120 cm. and of weight 4 newton , is hinged at A to a hinge fixed at a vertical wall. A weight of magnitude 3 newton is attaced to the rod at a point 40 cm. apart from B the rod is kept in a static equilibrium in a horizontal position by means of a string attached at the end B of the rod , its other end is fixed at a point C on the wall lying vertically above A such that AC = 160 cm.

Find the magnitude of the tension in the string and the magnitude and the direction of the reaction of the hinge.

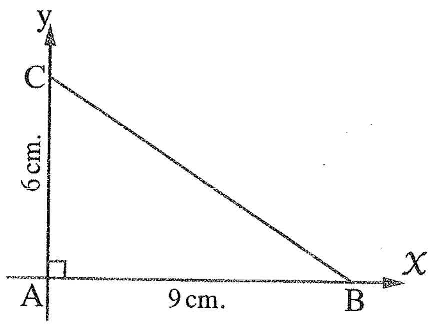
[b] A uniform rod rests with its upper end on a vertical wall ; the coefficient of friction between the rod and the wall is equal to $\frac{1}{2}$. If the rod rests with its lower end on a horizontal plane ; the coefficient of friction between the rod and the plane is equal to $\frac{3}{4}$. Find the tangent of the angle which the rod makes with the horizontal when it is about to slip.

4 ABCD is a square whose side length is 100 cm. , two forces of magnitudes 60 , 60 newton act in the direction of $\overrightarrow{BA}$, $\overrightarrow{DC}$. Find two forces equal in the magnitude , acting at A and C , parallel to $\overleftrightarrow{BD}$ and forming a couple equivalent to the couple formed by the first two forces.

5 ABCD is a rectangle in which AB = 9 cm. , BC = 24 cm. , E and F are midpoints of $\overline{BC}$ and $\overline{AD}$ respectively. The forces of magnitudes 18 , 48 , 30 and 24 gm. wt. act in the direction of $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CF}$ and $\overrightarrow{FA}$ respectively prove that the system is equivalent to a couple and find the norm of its moment , then find two forces acting along $\overrightarrow{EA}$, $\overrightarrow{FC}$ so that the system is in equilibrium.

6 **In the opposite figure :**

The centre of gravity of three equal masses each of magnitude = 2 kg. are fixed at the vertices of a right-angled triangle in which the lengths of the sides of its right angle are 6 cm. , 9 cm. is ....



(a) (2 , 3)

(b) (4.5 , 3)

(c) (3 , 2)

(d) (6 , 4)

7 The centre of gravity of the system consists of two masses 6 and 9 kg. and the distance between them is 10 m. lies at a distance .... metres from the first mass.

(a) 3

(b) 4

(c) 5

(d) 6

8 Four equal masses , each of magnitude 100 gm. are placed at the vertices of a square ABCD , determine the distance between the centre of gravity of the system and both $\overrightarrow{AB}$ and $\overline{AD}$

9 A body of weight 21 newton is placed on a rough horizontal plane , two horizontal forces act on the body of magnitudes 3 newton and 5 newton and include an angle of measure 60°. If the body is about to move , then the coefficient of static friction equals ....

(a) $\frac{3}{7}$

(b) $\frac{1}{7}$

(c) $\frac{1}{3}$

(d) $\frac{3}{5}$

10 If the limiting static friction force = 60 newton , the resultant reaction force equals 100 newton , then the normal reaction force = ... newton.

(a) 60

(b) 80

(c) 100

(d) 200

11 A body of weight (w) is placed on rough plane inclined to the horizontal at an angle of measure (θ). If the least force acting parallel to the line of the greatest slope of the plane to make the body about to move upwards the plane epuals ($2\ w \sin \theta$)

Prove that :

[a] The measure of the angle of friction = θ

[b] The magnitude of the resultant reaction force = w

12 If $\vec{F} = (2 , -3 , 4)$ acts at the point $(1 , 1 , 1)$, then the component of the moment of $\vec{F}$ about the X-axis equals ....

(a) 7

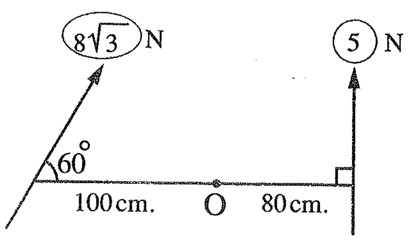
(b) – 2

(c) – 5

(d) 2

13 **In the opposite figure :**

The sum of the moments of the forces about the point O equals .... N. cm.



(a) 800

(b) – 800

(c) 400

(d) – 1200

14 *Answer one of the following items :*

[a] If the force $\overrightarrow{F} = 2\,\overrightarrow{i} - \overrightarrow{j} + 3\,\overrightarrow{k}$ acts at the point A (– 3 , 1 , 2) , find the moment vector of the force $\overrightarrow{F}$ about the point B (2 , 2 , – 1) , then calculate the length of the perpendicular drawn from the point B on the line of action of the force.

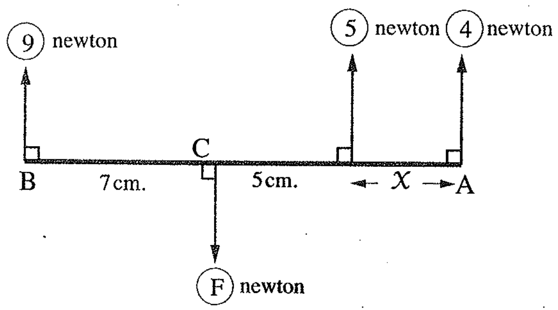
[b] The forces $\overrightarrow{F_1} = \ell\,\overrightarrow{i} + m\,\overrightarrow{j}$, $\overrightarrow{F_2} = \overrightarrow{i} - 3\,\overrightarrow{j}$, $\overrightarrow{F_3} = -2\,\overrightarrow{i} + \overrightarrow{j}$ act at the points A (1 , 2) , B (0 , 4) , C (2 , 4) respectively. If the sum of the moments of the forces about the origin point = – 9 $\overrightarrow{k}$ and the sum of the moments of the forces about the point D (– 2 , 3) equals – 4 $\overrightarrow{k}$ Find the value of each of ℓ and m

15 $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two parallel forces act in opposite directions. If $F_1 = 6$ newton , $F_2 = 8$ newton. If the distance between the second force and the resultant equals 15 cm. , then the distance between the two forces equals .... cm.

(a) 30
(b) 15
(c) 14
(d) 5

16 **In the opposite figure :**

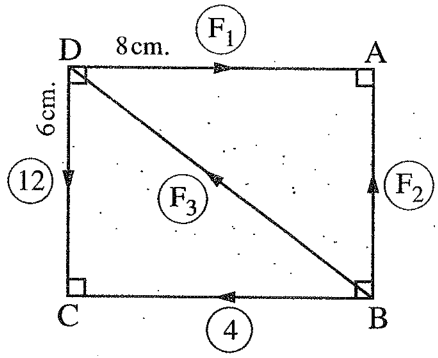
If $\overline{AB}$ is a rod in equilibrium horizontally , then the distance $X =$ .... cm.



(a) 9.5
(b) 14.5
(c) 4.5
(d) 18

17 **In the opposite figure :**

If the magnitude of the forces is in newton , the system is in equilibrium , then $F_1 =$ .... newton.



(a) 16
(b) 5
(c) 3
(d) 8

18 $\overline{AB}$ is a uniform rod of length 90 cm. and of weight 60 newton suspended horizontally at its two ends A and B by two vertical strings. Where should a weight of magnitude 150 newton be suspended in order that the magnitude of the tension at A is twice the magnitude of the tension at B ?

Egypt exam 1st session 2018 on statics

Answer the following questions :

1 A body of weight 36 newton is placed on a rough horizontal plane. If the coefficient of the static friction between the body and the plane equals $\frac{1}{3}$ and a horizontal force acts on the body trying to move it , then the magnitude of the friction force $\in$ ....

(a) $]\frac{1}{3} , 12]$

(b) $]\frac{1}{3} , 36]$

(c) $]0 , 12]$

(d) $]0 , 36]$

2 If a set of forces are in equilibrium , then ....

(a) Only the sum of the moments of the forces about any point vanishes.

(b) Only the resultant of the forces vanish.

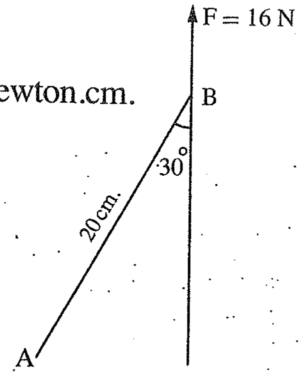
(c) The sum of the moments of the forces about any point vanishes and the resultant of the forces vanish.

(d) The resultant of the forces equals the sum of the magnitudes of the forces and the sum of the moments of the forces about any point are not vanishes.

3. A , B , C and D are four different points lying on a straight line where : AB = BC = CD = 30 cm. Two forces of magnitudes 8 , 9 Newton act at the points A and D respectively and in the same direction perpendicular to the straight line. Another two forces of magnitudes 4 , 7 Newton act at the points B and C respectively in the opposite direction of the first two forces. Find the resultants of these forces and the distance between the point of action of the resultant and A

4. $\overline{AB}$ is a rod of length 50 cm. and weighs 20 Newton, acts at its midpoint. The rod can rotate easily in a vertical plane about a fixed hinge at its end A. If a couple of moment 250 newton.cm. acts on the rod in a vertical plane. Find the reaction of the hinge and the inclination angle of the rod to the vertical in the equilibrium position.

5 **In the opposite figure :**



If F = 16 newton , then the moment of F about A equals .... newton.cm.

(a) 320

(b) $160\sqrt{3}$

(c) 160

(d) – 320

6 If $\overrightarrow{F_1} = 6\hat{i} + b\hat{j}$, $\overrightarrow{F_2} = a\hat{i} - 4\hat{j}$, are the two forces of a couple , then a + b = ...

(a) 10

(b) – 10

(c) – 2

(d) 2

7 A uniform rod of length 4 metres and weight 50 kg.wt. rests horizontally on two supports at its ends. If a weight of magnitude 20 kg.wt. is fixed at 1 metre a part from one of its ends , find the reaction of the two supports.

8 ABCD is a rectangle in which AB = 30 cm. , BC = 40 cm. forces of magnitudes 15 , 30 , 15 and 30 dyne act along $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$ and $\overrightarrow{DA}$ respectively. Prove that this system is equivalent to a couple and find its moment , then find the two forces acting at A and C perpendicular to $\overline{AC}$ such that the system is in equilibrium.

9 If the force $\vec{F} = 2\hat{i} - \hat{j} + 5\hat{k}$ acts at the point A (3 , –1 , 4) , then the component of the moment of $\vec{F}$ about the X-axis equals ....

(a) – 1

(b) 1

(c) – 9

(d) 9

10 The centre of gravity of a system made up of two masses 3 kg. and 5 kg. the distance between them is 8 metres is at a distance of .... metres from the first mass.

(a) 3

(b) 4

(c) 5

(d) 6

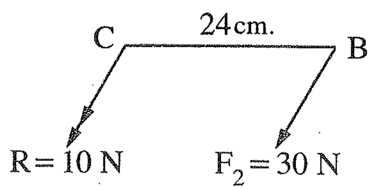
11 ***Answer one of the following items :***

[a] If the force $\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$ acts at the point A (1 , – 1 , 4) find the moment of the force $\vec{F}$ about the point B (2 , – 3 , 1) , then determine the length of the perpendicular drawn from the point B on the line of action of the force $\vec{F}$.

[b] ABCD is a trapezium in which , m (∠ ABC) = m (∠ BDC) = 90° , $\overline{AD} // \overline{BC}$, AB = 12 cm. , BC = 25 cm. and AD = 9 cm. Forces of magnitudes 75 , F , 50 newton act at $\overrightarrow{DA}$, $\overrightarrow{BA}$ and $\overrightarrow{DB}$ respectively. If the algebraic sum of the moments of these forces about the point C vanishes , find F and the algebraic sum of the moments of these forces about the point E such that E ∈ $\overline{BC}$, BE = 5 cm.

12 **In the opposite figure :**

If $\overrightarrow{F_1} // \overrightarrow{F_2}$ and act at A , B respectively such that A ∈ $\overleftrightarrow{BC}$, BC = 24 cm. , then AB = ... cm.



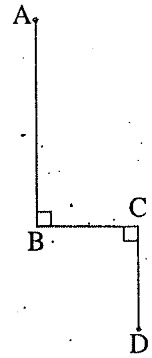
(a) 6

(b) 12

(c) 18

(d) 48

13 **In the opposite figure :**



ABCD is a wire in which AB = 2 BC = 2 CD = 16 cm.

, then the coordinates of the centre of gravity of the wire about each of $\overleftrightarrow{BC}$ and $\overleftrightarrow{BA}$ respectively is ....

(a) (3 , 3)

(b) (4 , 4)

(c) (3 , 5)

(d) (4 , 8)

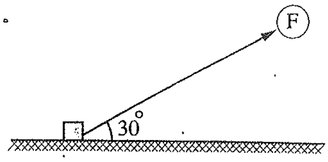
14 ***Answer one of the following items :***

[a] A uniform ladder rests in its final equilibrium with its upper end on a rough vertical wall and with its lower end on a horizontal rough ground. If the coefficients of static friction between the ladder and each of the wall and the ground equals $\frac{2}{3}$, $\frac{1}{4}$ respectively , find the measure of the angle of inclination for the ladder to the ground.

[b] A uniform rod of weight (w) is attached at one of its ends by a hinge and the other end is attached by a string joined to a point at the same horizontal plane passing through the hinge such that the measure of the angle of inclination for each of the rod and the string to the horizontal is equal to θ. Prove that the reaction at the hinge is equal to $\frac{w}{4}\sqrt{\cot^2\theta + 9}$

15 **In the opposite figure :**

A body of weight 48 Newton is placed on a rough horizontal plane and the measure of the angle of friction between the body and the plane equals 60°. If a force inclined to the horizontal at an angle of measure 30° acts on the body to make it about to move on the plane , then the magnitude of the force equals .... newton.



(a) 48

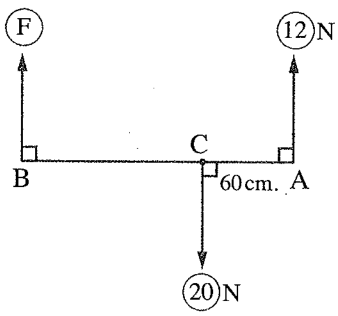
(b) 24

(c) 36

(d) 12

16 **In the opposite figure :**

If the system of coplanar forces are equilibrium , then BC = ... cm.



(a) 45

(b) 150

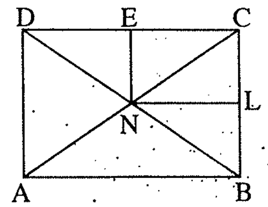
(c) 90

(d) 8

17 A body of weight 80 Newton is placed on a rough plane inclined to the horizontal at an angle of measure θ such that $\tan\theta = \frac{3}{4}$ and the coefficient of the static friction between the body and the plane equals μ_s

If a horizontal force of magnitude 160 newton acts on the body to make it about to move upwards the plane , find the value of μ_s

18 **In the opposite figure :**



ABCD is a lamina of a uniform thickness and density in the form of a rectangle in which AB = 12 cm. , BC = 8 cm.

If L , E are the midpoints of $\overline{BC}$, $\overline{CD}$ respectively , $\overline{AC} \cap \overline{BD} = \{N\}$

The rectangle NLCE is cut off from the lamina. Find the distance between the centre of gravity of the remaining part of the lamina in this case and both $\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$

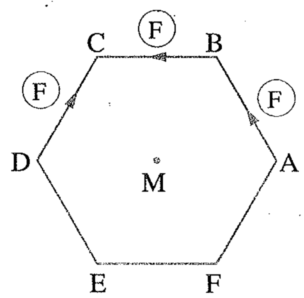
If the lamina is suspended freely from A , find the tangent of the inclination angle of $\overline{AB}$ to the vertical in the equilibrium position.

Egypt exam 2nd session 2018 on statics

Answer the following questions :

1 In the opposite figure :

ABCDEF is a regular hexagon whose side length is (ℓ) , three equal forces each of magnitude F act at $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$ respectively , then the algebraic sum of the moments of these forces about the point M (the centre of the hexagon) equals .... moment unit.



(a) $\frac{3\sqrt{3}}{2} F \ell$

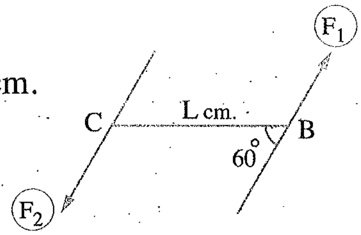
(b) $\frac{\sqrt{3}}{3} F \ell$

(c) $\frac{\sqrt{3}}{2} F \ell$

(d) $\frac{-\sqrt{3}}{2} F \ell$

2 In the opposite figure :

$F_1 = 7$ newton , the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ form a couple whose moment equals 210 newton.cm. , then L = ... cm.



(a) 30

(b) $30\sqrt{3}$

(c) $20\sqrt{3}$

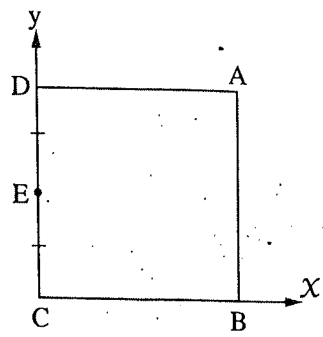
(d) $15\sqrt{3}$

3 If the forces $\overrightarrow{F_1} = 2\hat{i} - 4\hat{j}$, $\overrightarrow{F_2} = \hat{i} - 3\hat{j}$, $\overrightarrow{F_3} = -3\hat{i} + 7\hat{j}$ act at the points A (– 1 , 1) , B (–2 , 3) , C (0 , 1) respectively , prove that the system of forces is equivalent to a couple and find its moment.

4 **In the opposite figure :**

ABCD is a square of side length 40 cm. , masses of magnitudes 5 , 10 and 15 kg. are attached at vertices A , B and C respectively. Another mass of magnitude 20 kg. is attached at E the midpoint of $\overline{CD}$

Identify the distance between the centre of gravity of the system and both $\overleftrightarrow{CB}$ and $\overleftrightarrow{CD}$. If the square is freely suspended from C , find the measure of the angle which $\overline{BC}$ makes with the vertical in the equilibrium position.



5 If the force $\vec{F} = 7\hat{j}$ acts at the point A (– 3 , 0), then the length of the perpendicular segment drawn from the point B (1 , – 2) to the line of action of the force $\vec{F}$ equals .... length unit.

(a) 4

(b) 7

(c) 28

(d) 2

6 The centre of gravity of the next system : $m_1 = 1$ kg. at (1 , 0) , $m_2 = 2$ kg. at (0 , 2) , $m_3 = 3$ kg. at (1 , 2) is ....

(a) $\left(\frac{-1}{3} , \frac{-1}{3}\right)$

(b) (2 , 1)

(c) $\left(\frac{5}{3} , \frac{2}{3}\right)$

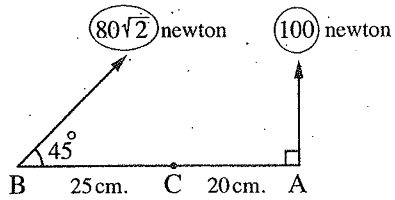
(d) $\left(\frac{2}{3} , \frac{5}{3}\right)$

7 *Answer one of the following items :*

[a] If the force $\overrightarrow{F} = 3\,\hat{i} - 2\,\hat{j} + 4\,\hat{k}$ acts at the point A (1 , 0 , – 1), find the moment of the force $\overrightarrow{F}$ about the point B (2 , – 1 , 3) , then determine the length of the perpendicular segment drawn from the point B on the line of action of the force $\overrightarrow{F}$

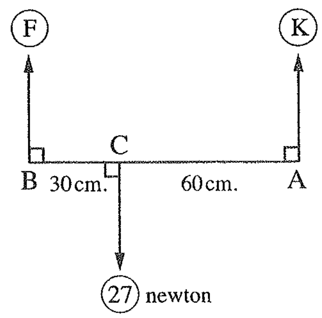
[b] In the opposite figure :

Prove that the line of action of the resultant of the two forces of magnitudes 100 newton and $80\sqrt{2}$ newton passes through the point C , then find the magnitude of the moment of the resultant of the forces about the point A



8 **In the opposite figure :**

If the system of coplanar forces are equilibrium , then F = ... newton



(a) 9

(b) 18

(c) 13.5

(d) 27

9 The centre of gravity of a system made up of two masses 7 kg. and 11 kg. distant 90 cm. from each other is distant ·············· cm. from the first mass.

(a) 50

(b) 55

(c) 35

(d) 45

10 ***Answer one of the following items :***

[a] A uniform rod rests in a vertical plane with its upper end on a smooth vertical wall and with its lower end on a horizontal rough ground. If the coefficients of static friction between the rod and the ground equals $\frac{1}{3}$, find the measure of the angle of inclination for the rod to the ground when it is about to slide.

[b] A uniform rod $\overline{AB}$ of 60 cm. length and weight 8 newton is hinged at its end A to a hinge fixed at a vertical wall. A weight of 6 newton is suspended at a point in the rod distant 40 cm. from the end A. The rod is being kept in a horizontal position by a light string attached at one of its two ends with the end B of the rod while the other end of the string is fixed at a point on the wall distant 80 cm. vertically upwards from A. Find the tension in the string and the reaction of the hinge.

11 If μ_s , μ_k are static and kinetic coefficient friction respectively of two bodies touch each other , then ....

(a) $\mu_s = \mu_k$

(b) $\mu_s < \mu_k$

(c) $\mu_s > \mu_k$

(d) There is no relation between them.

12 $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two parallel forces, the magnitude of the first equals 10 kg. wt. , and the magnitude of their resultant (R) equals 16 kg. wt. If the distance between $\overrightarrow{F_1}$, $\overrightarrow{R}$ equals 12 cm. , $\overrightarrow{F_1}$ and $\overrightarrow{R}$ work in the same direction, then the distance between the points of action of $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ equals .... cm.

(a) 8

(b) 16

(c) 20

(d) 32

13 A body of weight 40 newton is placed on a rough plane inclined to the horizontal at an angle of measure 30°. If a force F acts on the body in the direction of the line of the greatest slope of the plane upwards to make it about to move on the plane upwards. If the coefficient of the static friction between the body and the plane equals $\frac{\sqrt{3}}{2}$, find the value of F

14 If the two forces $\overrightarrow{F_1} = 3\hat{i} - \hat{j}$ and $\overrightarrow{F_2} = -9\hat{i} + 3\hat{j}$ act at the two points A (– 1 , 0) and B (1 , 2) respectively, find the resultant of the two forces and the coordinates of its point of effect.

15 A body of weight 40 newton is placed on a rough horizontal plane. If a horizontal force of magnitude 20 newton acts on it to make it about to move, then the magnitude of the resultant reaction force = ... newton

(a) $40\sqrt{5}$

(b) $20\sqrt{5}$

(c) $\frac{1}{4}\sqrt{5}$

(d) $\frac{1}{2}\sqrt{5}$

16 $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two parallel forces , where $7\overrightarrow{F_1} = 6\overrightarrow{F_2}$ and their resultant acts at a point at a distance 42 cm. far from the point of action of $\overrightarrow{F_2}$, then the distance between the line of action of the resultant and $\overrightarrow{F_1}$ equals .... cm.

(a) 78

(b) 36

(c) 49

(d) 6

17 $\overline{AB}$ is a uniform rod of length 4 metres and weight 10 kg.wt. rests horizontally on two supports the first at A and the second is at a distant 1 metre from B

Identify at which point on the rod a weight of magnitude 50 kg. wt. should be suspended in order that the magnitudes of the pressure on the two supports are equal.

18 ABC is a triangle in which AB = BC = 8 cm. , m (∠ ABC) = 120° , forces of magnitudes 12 , 12 , $12\sqrt{3}$ Newton act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ respectively.
Prove that this system is equivalent to a couple and find its moment.

Model examinations in the booklet form

Model 1

Answer the following questions :

1 The coefficient of friction between two bodies is determined due to .... of touching bodies.

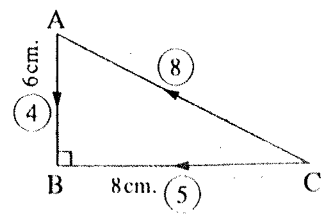
(a) form

(b) weight

(c) volume

(d) nature

2 **In the opposite figure :**



ΔABC is right-angled at B , forces of magnitudes 8 , 4 and 5 newton act along $\overrightarrow{CA}$, $\overrightarrow{AB}$ and $\overrightarrow{CB}$, then the norm of the sum of moments of the forces about A = ... newton.cm.

(a) 30

(b) 32

(c) 38.4

(d) zero

3 If $\overrightarrow{F_1} \parallel \overrightarrow{F_2}$ and they are in opposite directions , then $\overrightarrow{R}$ = ...

(a) $\overrightarrow{F_1} - \overrightarrow{F_2}$

(b) $\overrightarrow{F_1} + \overrightarrow{F_2}$

(c) $\overrightarrow{F_1} \times \overrightarrow{F_2}$

(d) $\overrightarrow{F_1} \cdot \overrightarrow{F_2}$

4 A rod is hinged to a vertical wall , X_1 , Y_1 are the algebraic components to the reaction of the hinge and if $X_1 = 3$ newton , $Y_1 = 4$ newton , then the reaction of the hinge equals .... newton.

(a) 1

(b) 5

(c) 7

(d) 12

5 The centre of gravity of a system made up of two masses 6 kg. and 9 kg. and the distance between them is 10 m. is distant .... m. from the first mass.

(a) 3
(b) 4
(c) 5
(d) 6

6 If $\overrightarrow{F} = 2\hat{i} + \ell\hat{j} - \hat{k}$ acts at the point A (4 , − 2 , 0) and the moment of $\overrightarrow{F}$ about the origin point is equal to $2\hat{i} + 4\hat{j} + 16\hat{k}$, then ℓ = ...

(a) 2
(b) 3
(c) 4
(d) 5

7 A light scaled ruler is suspended horizontally by two vertical strings , one of them is at a scale 10 and the other at the scale 70 , a weight 12 kg.wt is suspended at the scale 25 , then find the tension in each string.

8 $\overline{AB}$ is a uniform ladder of length 8 metres and of weight 20 kg.wt. rests at its end A on a rough horizontal ground and rests at one of its points C on the edge of a smooth fence of height 4 metres above the ground. If the ladder is about to slide when it inclines to the horizon with an angle of tangent $\frac{4}{3}$, find the coefficient of friction between the ladder and the ground.

9 A uniform square lamine of weight 40 gm.wt. is suspended freely from the vertex A and a weight of 10 gm.wt. is fixed at the vertex B Find the measure of the angle of inclination of the diagonal $\overline{AC}$ to the vertical in the equilibrium position.

10 ***Answer one of the following items :***

[a] If $\vec{F} = \ell \hat{i} + m \hat{j}$ act at the point A (4 , 2) and the moment vector of $\vec{F}$ about the origin point is $-22 \hat{k}$ and about the point B (13 , 1) is $22 \hat{k}$ Find the values of ℓ , m

[b] If $\vec{F} = \hat{i} - 2\hat{j}$ acts at a point A (2 , 3) Find the length of the perpendicular drawn from B (2 , 1) to the line of action of the force $\vec{F}$

11 Two parallel forces are in opposite directions , their magnitudes 5 and 8 newton act at the two points A and B where AB = 24 cm. , then the distance between the point of the action of the resultant and the second force = ... cm.

(a) 20

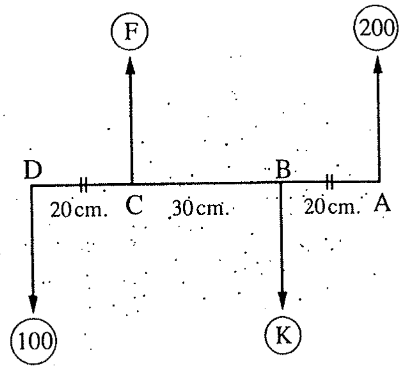
(b) 40

(c) 60

(d) 80

12 The opposite figure shows the parallel forces measured by newton act on a rod $\overline{AD}$, if the magnitude of the resultant is 300 newton and it acts upwards at 40 cm. from A and lies between A and D

Find : F and k



13 ABCD is a square of side length 16 cm. Forces of magnitudes 40 , F , 40 , F gm.wt. act along $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$, $\overrightarrow{DA}$ respectively. If these four forces are equivalent to a couple the norm of its moment equals 480 gm.wt.cm. in the cyclic direction ADCB **Find :** F

14 ***Answer one of the following items :***

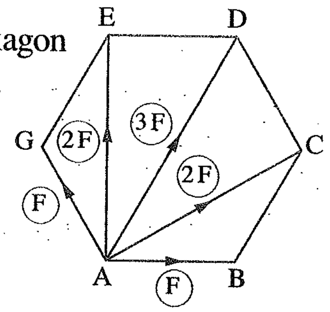
[a] A body of weight 30 newton is placed on a rough inclined plane. When the plane is inclined at 30° to the horizontal , the body is about to slide down.

If the inclination of the plane to the horizontal is increased to be 60° calculate the magnitude of the force act on the body and is parallel to the line of greatest slope to make the body about to move upwards.

[b] A boy pushes a stone of weight 56 newton by a horizontal force of magnitude 42 newton on a ridge , the stone was about to move. Find the coefficient of static friction between the stone and the ridge.

15 **In the opposite figure :**

If the sum of moments of the forces act on the opposite regular hexagon vanishes about a point on the plane such as N , then N ∈ ....



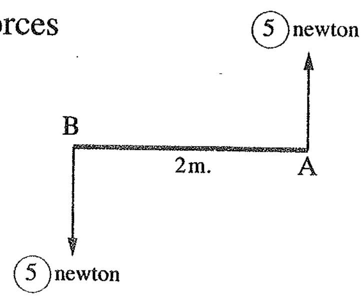
(a) $\overline{AG}$

(b) $\overline{AC}$

(c) $\overline{AD}$

(d) $\overline{AE}$

16 The algebraic measure of the moment of the couple of the forces shown in the opposite figure measured by newton. m. equals ....



(a) – 50

(b) – 10

(c) 10

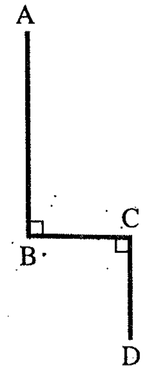
(d) 50

17 **In the opposite figure :**

ABCD is a wire of length 32 cm.

in which AB = 2 BC = 2 CD = 16 cm.

, then the distance between the centre of gravity of the wire and both $\overleftrightarrow{BC}$ and $\overleftrightarrow{BA}$ respectively is ....



(a) (3 , 3)

(b) (4 , 4)

(c) (3 , 5)

(d) (4 , 8)

18 ABCD is a parallelogram , in which AB = 18 cm. , BC = 20 cm. , m (∠ A) = 30° forces of magnitudes 8 , 6 , 8 , 6 newton act along $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$ and $\overrightarrow{DA}$ respectively. Prove that this system is equivalent to a couple , then find the norm of its moment , then find the magnitude of each of the two forces which act at A and D and are perpendicular to $\overline{AD}$ and equivalent to the previous system.

Model 2

Answer the following questions :

1. Two parallel forces , are in the same direction and of magnitudes 6 , 8 N , then the magnitude of their resultant = ... N

   (a) 2
   (b) 10
   (c) 14
   (d) 48

2. The ratio of the magnitude of the limiting static friction to the normal reaction is called ....

   (a) the force of static friction.
   (b) the coefficient of static friction.
   (c) the resultant reaction.
   (d) the angle of friction.

3. If the sum of the moments of the forces about A = – the sum of the moments of the forces about B , then the line of action of the resultant is ....

   (a) perpendicular to $\overline{AB}$
   (b) parallel to $\overline{AB}$
   (c) passing through the midpoint of $\overline{AB}$
   (d) coincide with $\overline{AB}$

4. The centre of gravity of the following system $m_1 = 1$ at (0 , 0) , $m_2 = 1$ at (3 , 0) , $m_3 = 2$ at (3 , 4) is ....

   (a) $\left(\frac{9}{4} , 0\right)$
   (b) $\left(\frac{9}{4} , 2\right)$
   (c) $\left(2 , \frac{9}{4}\right)$
   (d) (9 , 8)

5 Two forces form a couple , the magnitude of one of the two forces is 13 newton and the moment of the couple is 65 newton. cm. , then the perpendicular distance between them equals .... cm.

(a) 5
(b) 52
(c) 78
(d) 845

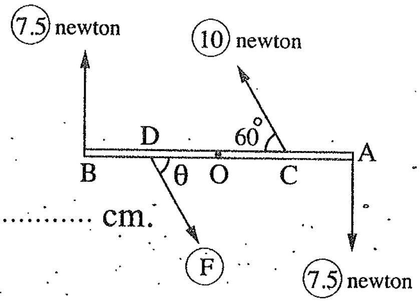
6 $\overline{AD}$ is a non-uniform rod rests on two supports at B and C where AB = BC = CD , it is found that the rod is about to rotate about B if a weight 5 kg.wt. is suspended at A and the rod is about to rotate about C if a weight 10 kg.wt. is suspended at D Find the weight of the rod.

7 ABCD is a square with side length 10 cm. , a force of magnitude 20 newton acts along $\overrightarrow{AB}$, then the norm of the moment of this force about the center of the square equals ....

(a) $50\sqrt{2}$
(b) 100
(c) $100\sqrt{2}$
(d) 200

8 **In the opposite figure :**

If $\overline{AB}$ is a light horizontal rod of negligible weight and its length is 30 cm. , O is the midpoint of $\overline{AB}$, then the length of $\overline{CD}$ in case of equilibrium equals .... cm.



(a) 10

(b) 15

(c) $10\sqrt{3}$

(d) $15\sqrt{3}$

9 When a weight W is placed on a rough inclined plane at an angle θ to the horizontal , it is found that the weight is about to slide down. Prove that the least force along the line of the greatest slope which makes the weight about to move upwards is equal to 2 W sin θ

10 A body of weight 39 newton is placed on a horizontal rough plane and if the tangent of the angle of friction between the body and the plane equals $\frac{1}{3}$, the body is pulled by a force inclines to the horizontal plane with an angle whose sine is $\frac{4}{5}$ that makes the body about to move , then the magnitude of the friction force = ... newton.

(a) 13

(b) 9

(c) 8

(d) 15

11 Three forces $\overrightarrow{F_1}$; $\overrightarrow{F_2}$, $\overrightarrow{F_3}$ act at the point (2 , 3) if $\overrightarrow{F_1} = 2\hat{i} + 4\hat{j}$, $\overrightarrow{F_2} = \hat{i} - 2\hat{j}$, $\overrightarrow{F_3} = -3\hat{i} + 4\hat{j}$, then the sum of the moments of these forces about the origin point equals ....

(a) $6\hat{k}$

(b) $12\hat{k}$

(c) $-6\hat{k}$

(d) $-12\hat{k}$

12 $\overline{AB}$ is a uniform ladder of weight (w) kg.wt. rests at its end A on a rough horizontal ground and it rests at the end B against a rough vertical wall such that the ladder is in a vertical plane perpendicular to the wall and it is inclined at the horizontal with an angle of measure 45° , if the coefficient of friction between the ladder and the ground equals $\frac{3}{7}$, then find the coefficient of friction between the ladder and the wall if the ladder is about to move.

13 ABCD is a square , its side length 10 cm. , forces of magnitudes 4 , 9 , 4 , 9 newton act along $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$ respectively , two other forces act at A and C , the magnitude of each is $5\sqrt{2}$ newton in directions $\overrightarrow{BD}$, $\overrightarrow{DB}$ respectively prove that the system is equivalent to a couple and find the norm of its moment.

14 ***Answer one of the following items :***

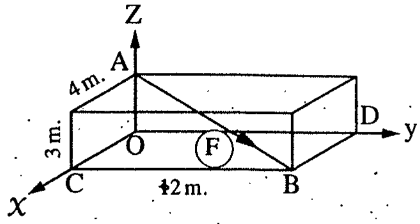
[a] The magnitude of the resultant of two parallel forces is 30 newton and the magnitude of one of the forces is 50 newton and acts at a distance of 12 cm. from the resultant find the distance between the lines of action of the two forces if the known force and the resultant have opposite direction.

[b] Two like forces of magnitudes F and 2 F act at the two points A and B. If the force 2 F moves parallel to itself in the direction of $\overrightarrow{AB}$ a distance x cm. , prove that the resultant of the two forces moves in the same direction a distance $\frac{2}{3}x$

15 **In the opposite figure :**

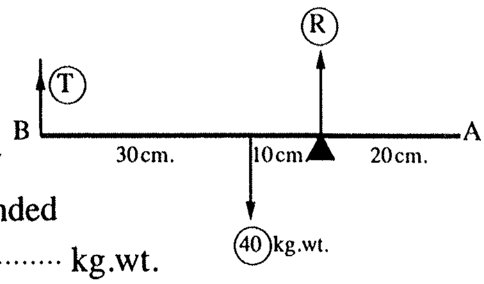
A force of a magnitude 130 newton acts along the diagonal $\overline{AB}$ in the cuboid whose dimensions are 3 m. , 4 m. and 12 m.

Find the moment of the force $\vec{F}$ about point D



16 **In the opposite figure :**

$\overline{AB}$ is a uniform rod of length 60 cm. and of weight 40 kg.wt. , if the rod rests horizontally on a support at a distance 20 cm. from A and suspended from the end B by a light string , then R – T = ... kg.wt.



(a) 40

(b) 30

(c) 10

(d) 20

17 ABCDEO is a regular hexagon. Forces of magnitudes $10\sqrt{3}$, 6, $10\sqrt{3}$, 6 newton act along $\overrightarrow{AB}$, $\overrightarrow{DB}$, $\overrightarrow{DE}$, $\overrightarrow{AE}$ respectively. Prove that the system tends to a couple and find the magnitude of its moment. Find also the magnitude and the direction of the two forces act along $\overrightarrow{EB}$, $\overrightarrow{AO}$ such that the system will be in equilibrium.

18 ***Answer one of the following items :***

[a] A fine lamina of uniform thickness and density in the form of a circular disc whose centre is the origin point and radius length is 24 cm. , two circular discs the centre of one to them is (– 2 , – 12) and radius length is 4 cm. where the centre of the other disc is (6 , 10) and the radius length is 12 cm. are cut off. Find the centre of gravity of the remaining part of the disc.

[b] A uniform squared lamina of weight (W) is suspended freely from the vertex A and a weight of $\left(\frac{1}{4}\text{ W}\right)$ is fixed at vertix B. Prove that the tangent of the angle of inclination of the diagonal $\overline{AC}$ to the vertical in the equilibrium position is equal to $\frac{1}{5}$

Model 3

Answer the following questions:

1. A body of weight 12 newton is placed on a horizontal rough plane , then the body is pulled by a horizontal force of magnitude $4\sqrt{3}$ newton which makes the body about to move , then the measure of the angle of friction between the body and the plane = ...

   (a) 30°
   (b) 45°
   (c) 60°
   (d) 75°

2. If $\vec{F} = (-1 , 3 , -2)$ acts at the point $(4 , -1 , 0)$, then the componant of the moment of $\vec{F}$ about z-axis equals ....

   (a) – 8
   (b) 3
   (c) 11
   (d) 13

3. Two parallel forces are in opposite directions and of magnitudes 5 and 12 newton , then the magnitude of their resultant = ... N

   (a) 7
   (b) 13
   (c) 17
   (d) 60

4. The centre of gravity of a uniform fine lamina on the shape of triangle ABC where A (1 , 2) , B (– 1 , 0) , C (3 , 1) is the point ....

   (a) (1 , 1)
   (b) (0 , 0)
   (c) (2 , 2)
   (d) (3 , 3)

5. Two parallel forces are in the same direction and of magnitudes 7 and 10 newton act at the two points A and B where AB = 51 cm. If their resultant acts at the point C , then AC = ... cm.

(a) 12

(b) 21

(c) 27

(d) 30

6. If μ_s , μ_k are static and kinetic coefficient friction respectively of two bodies touch each other , then ....

(a) $\mu_s = \mu_k$

(b) $\mu_s < \mu_k$

(c) $\mu_s > \mu_k$

(d) There's no relation between them.

7. A uniform ladder of weight 16 kg.wt. and of length (2ℓ) m. rests with one of its two ends against a smooth vertical wall and with the other end on a rough horizontal ground such that it lies on a vertical plane perpendicular to the wall and inclined to the horizontal with an angle of measure 45° , if the coefficient of friction between the ladder and the ground equals 0.75 , then determine the horizontal force acting at the lower end of the ladder so that the motion is about to begin a way from the wall.

8 ***Answer one of the following items :***

[a] Two parallel forces of the same directions act at the two points A and B where AB = 40 cm. , if the magnitude of their resultant 25 gm.wt. and acts at the point $C \in \overline{AB}$ where AC = 16 cm. Find the magnitude of the smallest force.

[b] ABC is a right-angled triangle at B in which AB = 6 cm. , BC = 8 cm. , the force $\vec{F}$ acts in the plane of the triangle such that $\mu_A = \mu_B = 60$ newton.cm. , $\mu_C = -60$ newton.cm. Find the magnitude of $\vec{F}$ and determine its line of action.

9 If the two forces $\overrightarrow{F_1} = 6\hat{i} - b\hat{j} + 9\hat{k}$ and $\overrightarrow{F_2} = 2a\hat{i} - 4\hat{j} + 3c\hat{k}$ form a couple , then (a , b , c) is ....

(a) (– 3 , – 4 , – 3)

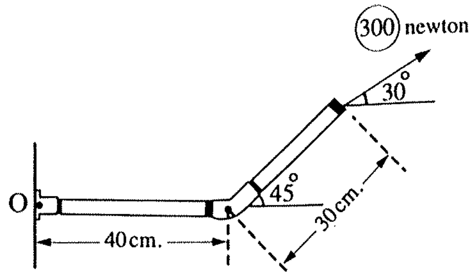
(b) (3 , 4 , 3)

(c) (3 , – 4 , – 3)

(d) (– 3 , 4 , 3)

10 **In the opposite figure :**

Find the algebraic measure of the moment of the force 300 newton about the point O (0 , 0)



11. A body of weight 3 newton is placed on a plane inclined at 30° to the horizontal and the coefficient of static friction between the weight and the plane is $\frac{2}{3}$, A force of magnitude 2 newton is acting on the body upwards along the line of the greatest slope. Given that the body is at rest , find the force of friction and investigate whether or not the motion is about to begin.

12. A uniform rod of length 180 cm. and weight 60 newton is suspended in a horizontal position at its two ends A and B by two vertical strings , then a body of weight 150 newton is suspended at the point C on the rod if the magnitude of the tension in the string at A is twice the magnitude of the tension in the string at B **Find :** AC

13. If the vector of the resultant of a set of forces is $\overrightarrow{R}$ and the vector of sum of the moments of the forces about one point in their plane is $\overrightarrow{M}$, then the condition of equilibrium of a set of coplanar forces is ....

(a) $\overrightarrow{R} = \overrightarrow{O}$, $\overrightarrow{M} = \overrightarrow{O}$

(b) $\overrightarrow{R} \neq \overrightarrow{O}$, $\overrightarrow{M} = \overrightarrow{O}$

(c) $\overrightarrow{R} = \overrightarrow{O}$, $\overrightarrow{M} \neq \overrightarrow{O}$

(d) $\overrightarrow{R} \neq \overrightarrow{O}$, $\overrightarrow{M} \neq \overrightarrow{O}$

14 ABC is an equilateral triangle of side length 8 cm. a force of magnitude 15 newton acts at $\overrightarrow{BC}$, then the norm of the moment of this force about the point A equals ....

(a) $40\sqrt{3}$

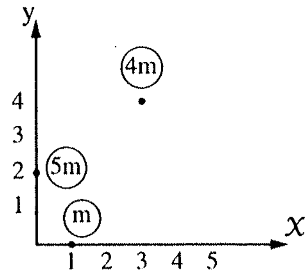
(b) 60

(c) $60\sqrt{3}$

(d) 120

15 **In the opposite figure :**

Three masses m , 4 m , 5 m , then the centre of gravity of the system lies on the point ....



(a) $\left(\frac{13}{10}, \frac{26}{10}\right)$

(b) $\left(\frac{18}{10}, \frac{37}{10}\right)$

(c) $\left(\frac{17}{10}, \frac{27}{10}\right)$

(d) $\left(\frac{26}{10}, \frac{13}{10}\right)$

16 ABC is a fine lamina of uniform thickness and density in the form of a right-angled triangle at B where AB = 12 cm. , BC = 20 cm. and X , Y and Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively. The triangle CYZ is cut off and coincided with the triangle YBX if the system is freely suspended from point B. Find the tangent of the angle of inclination of $\overline{BC}$ to the vertical in the equilibrium position.

17 *Answer one of the following items :*

[a] ABCD is a rectangle in which AB = 120 cm. , BC = 50 cm. , forces of magnitudes 50 , 10 , 50 , 10 newton act along $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$ respectively find two forces act at B and D perpendicular to $\overline{BD}$ such that the system is in equilibrium.

[b] Forces $\overrightarrow{F_1} = 2\hat{i} - 4\hat{j}$, $\overrightarrow{F_2} = \hat{i} - 3\hat{j}$, $\overrightarrow{F_3} = -3\hat{i} + 7\hat{j}$ act at the points A (– 1 , 1) , B (– 2 , 3) , C (0 , 1) respectively prove that this system is equivalent to a couple and find its moment norm.

18 ABC is a triangle in which AB = AC = 13 cm. , BC = 24 cm. forces of magnitudes 39 , 72 , 39 newton act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ respectively prove that the system is equivalent to a couple and find the norm of its moment.

Model 4

Answer the following questions :

1 Two parallel forces are in opposite directions , the magnitude of one of them is 7 newton and the magnitude of their resultant is 10 newton , then the magnitude of the other force = ... newton.

(a) 3
(b) 6
(c) 17
(d) 27

2 A body of weight 1 newton is placed on a horizontal rough plane , the coefficient of friction between it and the body is $\sqrt{3}$, then the resultant reaction force $\in$ ....

(a) $[0 , 1]$
(b) $[1 , 2]$
(c) $\{1 , 2\}$
(d) $\{2\}$

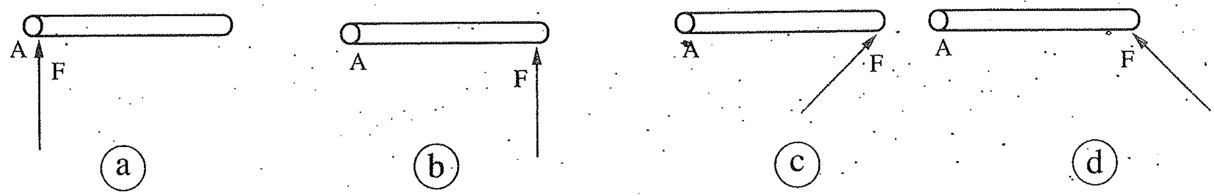
3 From the following sets of forces , there are two parallel forces act at two opposite directions , then they are ....

(a) $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{F_2} = 4\hat{i} - 6\hat{j}$
(b) $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{F_2} = -4\hat{i} + 6\hat{j}$
(c) $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{F_2} = 6\hat{i} - 4\hat{j}$
(d) $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$, $\overrightarrow{F_2} = -6\hat{i} + 4\hat{j}$

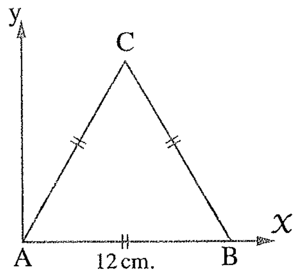
4 The two forces acting on the steering wheel of a car and producing the rotation of the steering wheel form ....

(a) a friction.
(b) an equilibrium.
(c) a couple.
(d) a perpendicular force on the steering wheel.

5 The following figures represent a door attached with a hinge at A If a force $\vec{F}$ acts on the door , in which of the following figures the force $\vec{F}$ has the greatest moment about A ?



6 The centre of gravity of the following system is at ....



Mass	4 mg.	5 mg.	3 mg.
Position	at A	at B	at C

(a) $\left(6.5 , \frac{3}{2}\sqrt{3}\right)$

(b) $\left(6.5 , 6\sqrt{3}\right)$

(c) $\left(6.5 , \frac{5}{2}\sqrt{3}\right)$

(d) $\left(9 , 3\sqrt{3}\right)$

7 If the force $\vec{F} = 3\hat{i} - 4\hat{j}$ acts at the point A (3 , – 1) , then the length of the perpendicular drawn from the point B (8 , – 4) to the line of action of this force equals ....

(a) 2

(b) 3

(c) 4

(d) 2.2

8 If a body is placed on an inclined rough plane with an angle of measure $\sin^{-1}\left(\frac{5}{13}\right)$ and it is about to move under the action of its weight , then the coefficient of static friction between the body and the plane = ...

(a) $\frac{5}{13}$

(b) $\frac{5}{12}$

(c) $\frac{12}{13}$

(d) $\frac{12}{5}$

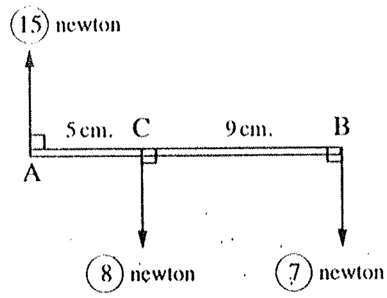
9 ***Answer one of the following items :***

[a] The magnitudes of the two parallel forces F , 36 newton and their resultant of magnitude 84 newton and acts in opposite direction of second force and at a distance 30 cm. from it find the distance between the two lines of action of the two forces.

[b] A 240 kg. weight is placed on a horizontal rough plane and is pulled by a string which makes an angle of 30° with horizontal. If the coefficient of static friction is 0.3 Calculate correct to two decimal places , the tension that makes the motion about to begin.

10 **In the opposite figure :**

The algebraic measure of the moment of the couple of set of forces which act on the rod $\overline{AB}$ measured by unit of newton. cm. equals ....



(a) – 138

(b) – 58

(c) 58

(d) 138

11 A uniform ladder of weight 15 kg. wt. and length (2ℓ) m. rests with one of its two ends on a rough horizontal ground and with the other end on a smooth vertical wall such that the ladder lies on a vertical plane perpendicular to the wall and it is about to slide when the measure of the inclination angle to the horizontal 45° , find the coefficient of friction between the ladder and the ground.

12 ***Answer one of the following items :***

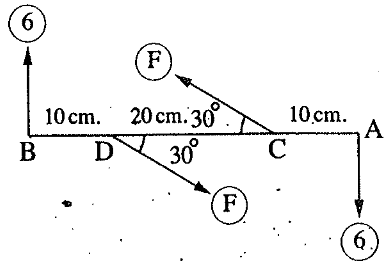
[a] A fine lamina of a uniform density bounded by the triangle ABC in which AB = BC = 9 cm. , m (∠ B) = 90° If the triangle ABM where M is the centre of gravity of the lamina is cut off and the remaining part is freely suspended from point B , find the tangent of the angle of inclination of $\overline{BC}$ to the vertical in the equilibrium position.

[b] ABCD is a square of side length 4 cm. The masses 6 , 4 , 3 and 2 gm. are attached at A , B , C and D respectively. Another mass of magnitude 10 gm. is attached at the midpoint of $\overline{AB}$ identify the distance between the centre of gravity of the system and both $\overline{CD}$ and $\overline{CB}$

13 $\overline{AB}$ is a uniform rod of length 60 cm. and of weight 30 newton , it is suspended in a horizontal position at its two ends by two vertical strings and carries two weights one of them is 10 newton , at a distant 10 cm. from the end A and the other 20 newton at 20 cm. distant from the end B , find the tension in each string.

14 **In the opposite figure :**

$\overline{AB}$ is a light rod of length 40 cm. If the rod is in equilibrium under the effect of the four forces find the value of F



15 A body of weight 38 kg. wt. is about to move under its own weight when placed on a rough plane which is inclined to the horizontal with an angle whose tangent is $\frac{1}{4}$

If the body is placed on a horizontal plane which is as rough as the inclined plane and is acted on it by an upward pull force in a direction inclined to the horizontal with an angle whose tan is $\frac{3}{4}$ so that the motion is about to begin. Find the magnitude of this force and the normal reaction.

16 If the mass 5 kg. acts at the point (2 , – 1) and the mass 7 kg. acts at the point (1 , 2) , then the centre of gravity of the two masses acts at the point ....

ⓐ (17 , 9)

ⓑ $\left(\frac{17}{12} , \frac{3}{4}\right)$

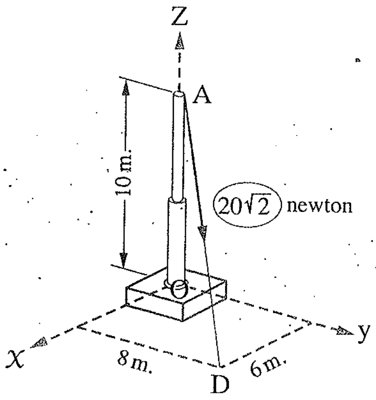
ⓒ (19 , 13)

ⓓ $\left(\frac{19}{12} , \frac{1}{4}\right)$

17 **In the opposite figure :**

A force of magnitude $20\sqrt{2}$ N.acts at point A

Find the moment of $\vec{F}$ about O



18 ABCD is a square of side length 3 cm. Forces of magnitudes 10 , 15 , 10 , 15 newton act along $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$ respectively also two equal forces of magnitude $5\sqrt{2}$ newton act on the two points A , C in the two directions $\overrightarrow{BD}$, $\overrightarrow{DB}$ respectively find the two forces acting at B and D parallels $\overline{AC}$ such that the system is in equilibrium.

Model 5

Answer the following questions :

1 The resultant reaction is the resultant of ....

(a) the weight of the body and the normal reaction.

(b) the weight of the body and limiting static friction force.

(c) the normal reaction and limiting static friction force.

(d) the kinetic friction force and the weight of the body.

2 If a force $\overrightarrow{F} = 2\hat{i} - \hat{j} + 5\hat{k}$ acts at the point A (1 , 0 , – 3) , then the moment of $\overrightarrow{F}$ about the point B which its position vector is $\hat{j} + 3\hat{k}$ equals ....

(a) $-2\hat{i} - 17\hat{j} + \hat{k}$

(b) $-11\hat{i} + \hat{k}$

(c) $-11\hat{i} - 17\hat{j} + \hat{k}$

(d) $-11\hat{i} - 17\hat{j}$

3 *Answer one of the following items :*

[a] The resultant of two parallel forces is 40 newton , one of the two forces 60 newton and acts at a distance 24 cm. far from the action line of the resultant. Find the distance between the two forces if the given force and the resultant act in the same direction.

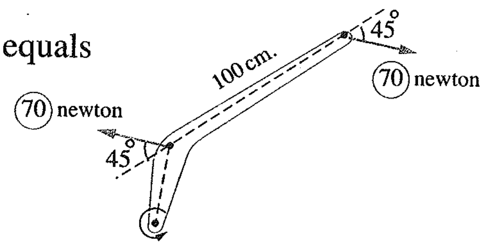
[b] If the force $\overrightarrow{F} = k\hat{i} + m\hat{j} - 2\hat{k}$ acts at point A whose position vector with respect to the origin point is $\overrightarrow{r} = (3 , 1 , 1)$ if the two components of the moment of $\overrightarrow{F}$ about x and y-axes are – 1 and – 8 respectively find the value of each of k and m.

4 A body lies under the action of the forces $\overrightarrow{F_1} = 2\hat{i} - a\hat{j}$, $\overrightarrow{F_2} = 5\hat{i} + 2\hat{j}$, $\overrightarrow{F_3} = b\hat{i} - 5\hat{j}$, if the body is in equilibrium , then (a , b) = ...

ⓐ (3 , 7)
ⓑ (– 3 , 7)
ⓒ (3 , – 7)
ⓓ (– 3 , – 7)

5 **In the opposite figure :**

The algebraic measure of the moment of the couple equals .... newton. cm.



ⓐ – 7000
ⓑ $-3500\sqrt{2}$
ⓒ $3500\sqrt{2}$
ⓓ 7000

6 $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two parallel forces where $3\overrightarrow{F_1} = 2\overrightarrow{F_2}$ and their resultant acts at a point at a distance 15 cm. far from the point of action of $\overrightarrow{F_1}$, then the distance between the line of action of the resultant and $\overrightarrow{F_2}$ = ... cm.

ⓐ 8
ⓑ 10
ⓒ 12
ⓓ 25

7 If the norm of the moment of a couple is 24 newton. m. and the arm length of the moment of the couple is 3 m. , then the magnitude of one of its two forces equals .... newton.

ⓐ 8
ⓑ 21
ⓒ 27
ⓓ 72

8. $\overline{AB}$ is a ruler of length 100 cm. and of weight (w) newton acting at its midpoint , it is suspended in a horizontal position by two vertical strings from its ends. At what point of the ruler a weight of (5 w) newton should be suspended so that the tension of one of the two strings twice the tension of the other string.

9. A uniform rod rests with its upper end a against a smooth vertical wall and its lower end on a horizontal plane , the coefficient of friction between it and the rod equals $\frac{1}{4}$ find the tangent of the angle which the rod makes with the horizontal when it is about to slide.

10 A uniform fine lamina is in the form of a rectangle in which AB = 18 cm. , BC = 24 cm. , the Δ ABE is separated where E is the midpoint of $\overline{AD}$, then the remaining part is suspended freely from C find the tangent of the angle of inclination of $\overline{CB}$ to the vertical in the equilibrium position.

11 A body is placed on a rough plane which is inclined to the horizontal with an angle of measure θ , the measure of the angle of friction is λ , then the body is still in equilibrium if and only if ....

(a) $\theta > \lambda$

(b) $\theta \geq \lambda$

(c) $\theta \leq \lambda$

(d) $\theta < \lambda$

12 ABCD is a rhombus of side length 12 cm. , m (∠ A) = 60° , forces of magnitudes 11 , 6 , 5 , 7 newton act along $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$, $\overrightarrow{DB}$ respectively , then the sum of algebraic magnitudes of the moments of these forces about A equals .... newton.cm.

(a) $12\sqrt{3}$

(b) $24\sqrt{3}$

(c) $36\sqrt{3}$

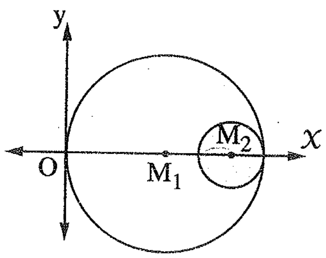
(d) $48\sqrt{3}$

13 ABCDE is a regular pentagon of side length 15 cm. Forces each of magnitude 10 kg.wt. act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$ and $\overrightarrow{EA}$ respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment.

14 $\overline{AB}$ is a rod of length 50 cm. and of weight 20 newton acting at its midpoint. It can rotate in a vertical plane about a fixed hinge at its end A , a couple of measure moment 250 newton. cm. acts in the vertical plane. Find the inclination angle of the rod to the vertical in the equilibrium position.

15 $\overline{AB}$ is a non uniform rod of length 30 cm. , rests horizontally on two supports at C and D where AC = CD = DB. If a weight of magnitude 6 kg.wt. is suspended at A , then the rod is about to rotate about C. and if a weight 9 kg.wt. is suspended at B , then the rod is about to rotate about D. Find the distance between the point of action of the weight of the rod and the end A

16 **In the opposite figure :**



A uniform fine circular lamina of radius length 30 cm. a circular disc of radius length 10 cm. is removed , and its centre is at a distance 20 cm. from the centre of the lamina , then the centre of gravity of the remaining part is at a distance .... cm. from the centre of gravity of the origianl disc.

(a) 1.5

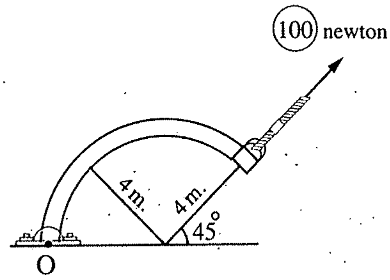
(b) 2

(c) 2.5

(d) 3

17 **In the opposite figure :**

The algebraic measure of the moment of the force 100 newton about (O) = ... newton.m.



(a) $100\sqrt{2}$

(b) 200

(c) $200\sqrt{2}$

(d) $400\sqrt{2}$

18 ***Answer one of the following items :***

[a] A body of weight (w) is placed on a rough inclined plane makes an angle of sine $\frac{5}{13}$ with the horizontal. The body is attached by a horizontal force of magnitude 22 newtons lies in the vertical plane which passes through the line of the greatest slope makes the body is about to move upwards the plane , if the static coefficient friction between the body and the plane is $\frac{1}{2}$, then find the magnitude of the weight (w).

[b] A body of weight 27 kg.wt. is placed on a horizontal rough plane , given that the coefficient of static friction between the body and the plane is $\frac{1}{3}$, find the magnitude of the force parallel to the plane which makes the body about to begin motion.

Model 6

Answer the following questions :

1. If the force $\overrightarrow{F} = 4\hat{i} - 3\hat{j}$ acts at the point (2 , 3) , then the vector moment of this force with respect to the point (– 1 , 2) equals ....

   (a) $-13\hat{k}$

   (b) $-5\hat{k}$

   (c) $5\hat{k}$

   (d) $13\hat{k}$

2. Two parallel forces of magnitudes 4 , 6 newton act in two opposite directions and the distance between their lines of action is 20 cm. , then the point of action of the resultant is at distance ....

   (a) 40 cm. from the first force.

   (b) 40 cm. from the second force.

   (c) 30 cm. from the first force.

   (d) 60 cm. from the second force.

3. The angle of friction is the included angle between ....

   (a) resultant reaction and limiting static friction.

   (b) resultant reaction and perpendicular reaction.

   (c) resultant reaction and weight of the body.

   (d) perpendicular reaction and force of kinetic friction.

4. If a rod rests at one of its ends on a rough plane , then the direction of the reaction of the plane ....

   (a) is perpendicular to the plane.

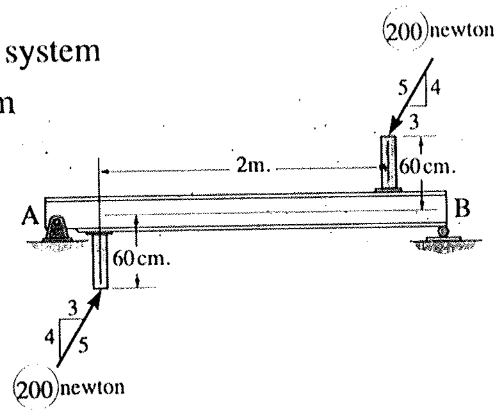
   (b) is parallel to the plane.

   (c) is unknown.

   (d) makes an angle of measure 45° with the plane.

5. ABCD is a fine uniform lamina in the shape of rectangle , its weight 4800 gm.wt. , AB = 6 cm. , BC = 8 cm. A weight 1200 gm.wt. is placed at the vertex B , determine the centre of gravity of the system and if the system is suspended freely from the vertex C , find in the equilibrium position the tangent of the angle of inclination of $\overline{CB}$ to the vertical.

6. The magnitude of the moment of the couple of the system of forces shown in the opposite figure in newton. m equals ....



(a) 144
(b) 176
(c) 320
(d) 464

7. If the force of limiting friction force 60 newton and coefficient of static friction is 0.75 , then magnitude of the resultant reaction equals .... newton.

(a) 60
(b) 80
(c) 100
(d) 200

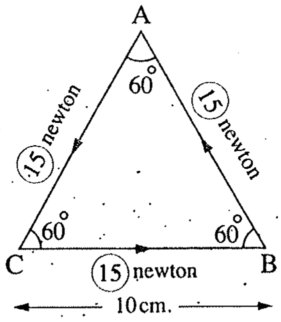
8 ***Answer one of the following items :***

[a] The magnitude of the smaller one of two parallel forces is 60 newton acts at the end A of a light rod $\overline{AB}$. While the larger one acts at the other end B. If the magnitude of their resultant is 20 newton and its line of action is at a distance of 120 cm. from the end B. What is the length of the rod $\overline{AB}$?

[b] A uniform rod of length 80 cm. and of weight 3 kg.wt. is suspended at its two ends in a horizontal position by two vertical strings , the maximum tension in any of them is 5 kg.wt. Find the positions of the point at which a mass 4 kg. can be suspended without cutting any of the two strings.

9 A uniform ladder of weight 32 kg.wt. and of length (2ℓ) rests in a vertical plane with one end against a smooth wall and the other end on a rough horizontal floor. and the inclination of the ladder to the floor is 45° and the magnitude of the smallest horizontal force acting at the lower end of the ladder so that the motion is about to begin away from the wall equals 4 kg.wt. Find the coefficient of friction between the ladder and floor.

10 The magnitude of the couple moment of the shown system of forces in the opposite figure in newton. cm. equals ....



(a) $37.5\sqrt{3}$

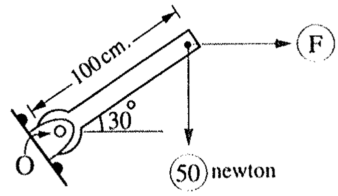
(b) $75\sqrt{3}$

(c) $100\sqrt{3}$

(d) $150\sqrt{3}$

11 **In the opposite figure :**

If the moment of the force F about "O" equals the moment of the force 50 newton about "O" , then F = ... newton.



(a) $50\sqrt{3}$

(b) $25\sqrt{3}$

(c) $12.5\sqrt{3}$

(d) 50

12 Two parallel forces of magnitudes F_1 , F_2 act in the same direction at the two points A and B respectively. If the force F_2 moves parallel to itself a distance = X on the ray $\overrightarrow{AB}$, prove that their resultant moves a distance equals $\frac{F_2}{F_2 + F_1} X$ in the same direction of moving.

13 A thin uniform wire is formed as a right-angled triangle at B in which AB = 3 cm. , BC = 4 cm. , then the distance between centre of gravity of the wire and each of $\overline{BA}$, $\overline{BC}$ is ....

(a) (1.5 , 1)

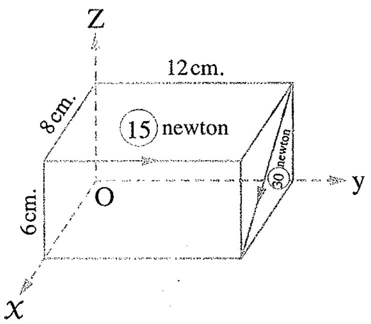
(b) (2 , 1.5)

(c) $\left(\frac{8}{7} , \frac{9}{14}\right)$

(d) $\left(\frac{12}{7} , \frac{11}{14}\right)$

14 In the opposite figure :

Find the sum of moments of the forces about O



(15) If R is the resultant of the two parallel forces 30 , F and R = 10 newton , then ....

(a) F = 20 newton and act in opposite direction of the force 30 newton.

(b) F = 20 newton and act in the same direction of the force 30 newton.

(c) F = 40 newton and act in opposite direction of the resultant.

(d) F = 40 newton and act in the same direction of the force 30 newton.

(16) ***Answer one of the following items :***

[a] A body of weight 25 kg.wt. is placed on a rough inclined plane and is acted on by a force P along the line of the greatest slope towards the top of the plane. Given that the motion is about to begin upwards when P = 15 kg.wt. , and it is about to begin downwards when P = 10 kg.wt. , find the angle of inclination of the plane to the horizontal.

[b] A body of weight W newton is placed on a horizontal rough plane where the measure of the angle of friction is λ , the body is pulled by a force making an angle of measure 2λ with the horizontal upwards , it makes the body about to begin motion. Prove that the magnitude of this force is $W \tan \lambda$

17 $\overline{AB}$ is a rod of length 100 cm. and of weight 3 kg.wt. acting at its midpoint. It can rotate in a vertical plane about a fixed hinge at its end A , a couple of magnitude moment 75 kg.wt. cm. acts in the vertical plane. Find the reaction of the hinge and the inclination angle of the rod to the vertical in the equilibrium position.

18 ABCD is a rectangle in which AB = 9 cm. , BC = 24 cm. , X and Y are the midpoints of $\overline{BC}$ and $\overline{AD}$ respectively , the forces of magnitudes 27 , 36 , 45 newton act along $\overrightarrow{AB}$, $\overrightarrow{BX}$, $\overrightarrow{XA}$ respectively. Find the two forces acting along $\overleftrightarrow{BY}$, $\overleftrightarrow{DX}$ to make the system in equilibrium.

Model 7

Answer the following questions :

1 Two parallel forces have the same directions , one of them is twice the other and the magnitude of their resultant equals 24 newton , then the magnitude of the great force in newton = ...

(a) 8

(b) 12

(c) 16

(d) 18

2 If the sum of moments of system of forces about A = sum of moments of the same system about B , then the line of action of the resultant ....

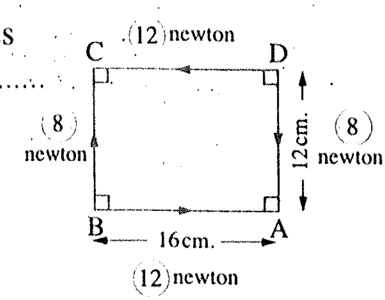
(a) is perpendicular to $\overline{AB}$

(b) is parallel to $\overline{AB}$

(c) bisects $\overline{AB}$

(d) is coincident on $\overline{AB}$

3 Algebraic measure of the moment of the couple of the forces shown in the opposite figure in the unit of newton. cm. = ...



(a) – 96

(b) – 16

(c) 16

(d) 96

4 F_1 and F_2 are two parallel forces acting at A , B respectively where $F_1 = 30$ kg.wt. , $F_1 > F_2$ and their resultant R of magnitude 10 kg.wt. and acts at C where BC = 90 cm. , then AB = ...

(a) 30 cm.

(b) 45 cm.

(c) 60 cm.

(d) 120 cm.

5. Force of magnitude 10 newton acts on $\overrightarrow{AB}$ where A (2 , 7) , B (5 , 3) , then the algebraic measure of its moment about the origin = ...

(a) 58

(b) 68

(c) – 58

(d) – 68

6. A , B , C , D are four collinear points such that AB = 2 BC = CD = 40 cm. parallel forces of magnitudes 20 , 30 , F_1 , F_2 act at the points A , B , C , D respectively such that the two forces 20 , F_2 act in one direction and opposite to the two forces 30 , F_1 and their resultant is 20 newton act in direction of F_1 and its line of action bisects $\overline{AD}$

Find F_1 , F_2

7. If $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are two forces form a couple acting at the two points A (1 , 1) , B (– 1 , 2) respectively where $\overrightarrow{F_1} = 2\hat{i} + 5\hat{j}$, then the moment of the couple equals ....

(a) $-12\hat{k}$

(b) $-\hat{k}$

(c) $\hat{k}$

(d) $12\hat{k}$

8 A uniform rod of weight w , rests in a vertical plane with one end against a smooth wall and the other end on a rough horizontal floor. If the inclination of the rod to the floor is 45° , prove that the coefficient of friction between the rod and the floor cannot have a value less than $\frac{1}{2}$

If the coefficient of friction is equal to $\frac{3}{4}$, find the horizontal force acting at the lower end of the rod so that the motion is about to begin towards the wall.

9 The distance between centre of gravity of a uniform thin lamina in the shape of an equilateral triangle whose side length is 12 cm. and one of the vertices of this triangle equals .... cm.

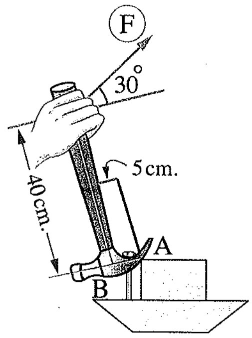
(a) $2\sqrt{3}$

(b) $4\sqrt{3}$

(c) 6

(d) $6\sqrt{3}$

10 The opposite figure illustrates the force F needed to remove a nail at B
If the magnitude of the moment of the force about point A needed to remove the nail is equal to 200 newtons. cm.
Find the magnitude of the force F



11 A body of weight 10 kg.wt. is placed on a rough horizontal plane. If the coefficient of static friction between the body and the plane is $\frac{1}{4}$ and a horizontal force of magnitude 2 kg.wt. act on it and if the magnitude of friction force is F, then ....

(a) F < 2 kg.wt.

(b) F = 2 kg.wt.

(c) 2 < F < 2.5 kg.wt.

(d) F = 2.5 kg.wt.

12. ABC is a triangle in which AB = 7 cm. , BC = 8 cm. , m (∠ ABC) = 120°. Forces of magnitudes 17.5 , 20 , 32.5 newton act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ respectively. Show that the system of these forces is equivalent to a couple , then find the magnitude of its moment.

13. ***Answer one of the following items :***

[a] A body of weight $2\sqrt{57}$ kg.wt. is placed on a rough horizontal plane. Two forces of magnitudes 4 and 6 kg.wt. including between them an angle of measure 60° acted on the body , the two forces are horizontal and lying in the same horizontal plane with the body. If the body became about to move. Find the coefficient of the static friction.

[b] A body of mass 10 kg. is placed on a rough plane inclined to the horizontal with an angle of measure 30° , the body becomes about to slide down. Find the force parallel to the plane which makes the body about to move upwards the plane.

(14) If the moment of the force $\vec{F} = 3\hat{i} + \hat{j} - 2\hat{k}$ about the point A is $\overrightarrow{M_A} = -3\hat{i} - \hat{j} - 5\hat{k}$, then the length of the perpendicular from this point to the line of action of the force equals .... length unit.

(a) $\frac{1}{5}\sqrt{10}$

(b) $\frac{1}{2}\sqrt{10}$

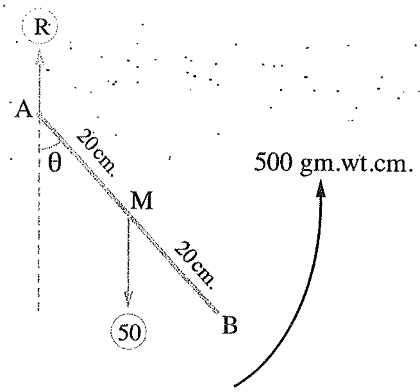
(c) $2\sqrt{10}$

(d) $10\sqrt{5}$

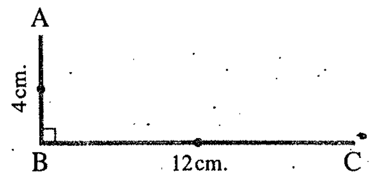
(15) A uniform rod $\overline{AB}$ of length 140 cm. and weight 16 newton is suspended by two vertical strings from its terminals, find the distance from A at which a weight of 14 newton can be suspended from one point on the rod to make the tension at A is twice the tension at B

(16) In the opposite figure :

$\overline{AB}$ is a uniform rod of length 40 cm. and of weight 50 gm.wt. it can rotate easily in a vertical plane about a hinge at the end A. When the rod was in vertical position, a couple of algebraic measure of moment = 500 gm.wt. cm. acts on the rod where the couple acts in the same vertical plane passing through the rod. Find in equilibrium position the measure of the angle with which the rod inclines to the vertical.



17 The opposite figure represents a wire of a uniform density and thickness such that AB = 4 cm. , BC = 12 cm. , m (∠ B) = 90° If the wire is suspended freely from B , then the tangent of the angle of inclination of $\overleftrightarrow{BC}$ to the vertical in the equilibrium position is ....



ⓐ $\frac{1}{9}$

ⓑ $\frac{1}{3}$

ⓒ $\frac{1}{2}$

ⓓ 3

18 ***Answer one of the following items :***

[a] Fine lamina of uniform thickness and density in the form of rectangle ABCD , where AB = 12 cm. , BC = 16 cm. , E is the intersection point of its diagonals $\overline{AC}$, $\overline{BD}$. The triangle AED is removed and placed on Δ BEC , find the centre of gravity of the lamina in this case with respect to the point C

[b] Where does the centre of gravity of the following system of 3 distributed masses lie ?

m_1 = 1 kg. at G_1 (0 , 0) , m_2 = 1 kg. at G_2 (3 , 0) , m_3 = 2 kg. at G_3 (3 , 4)

Model 8

Answer the following questions :

1 If $\overrightarrow{F} = (-1 , 3 , -2)$ acts on the point $(4 , -1 , 0)$, then the component of the moment of $\overrightarrow{F}$ about the z-axis equals ....

(a) -8

(b) 3

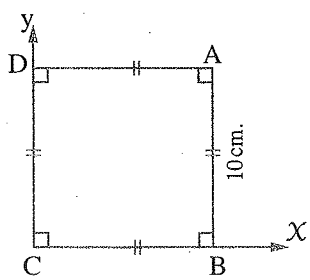
(c) 11

(d) 13

2 Two parallel forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, magnitude of their resultant 70 newton and magnitude of the first force 50 newton and its line of action at a distance 30 cm. from the line of action of the resultant. Determine magnitude and direction of the line of action of $\overrightarrow{F_2}$ if $\overrightarrow{F_1}$ and $\overrightarrow{R}$ have the same direction.

3 The centre of gravity of the following system is ....

Mass	20 gm.	30 gm.	10 gm.	40 gm.
Position	at A	at B	at C	at D



(a) $(4 , 7)$

(b) $(7 , 4)$

(c) $(5 , 6)$

(d) $(6 , 5)$

4 If the two forces $\overrightarrow{F_1} = 4\hat{i} - a\hat{j}$, $\overrightarrow{F_2} = 2b\hat{i} + 5\hat{j}$ form a couple , then $2a + b =$ ....

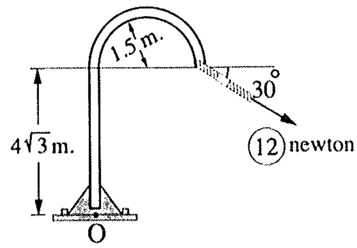
(a) -12

(b) -8

(c) 8

(d) 12

5 **In the opposite figure :**



The norm of the moment of the force whose magnitude is 12 newton with respect to the point "O" equals .... newton. cm.

(a) 30

(b) 60

(c) 90

(d) 190

6 Two parallel forces 20 , F newton , if the magnitude of their resultant is 35 newton and the distance between line of action of the given force and the resultant equals 15 cm. and the given force and resultant acting in opposite directions , then the distance between the point of action of the force F and point of action of the resultant equals ....

(a) $\frac{60}{11}$ cm.

(b) $\frac{105}{11}$ cm.

(c) $\frac{160}{11}$ cm.

(d) $\frac{225}{11}$ cm.

7 ABCD is a square whose diagonals intersect at M , forces of magnitudes 6 , F , k , 3 newton in the directions $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$ respectively if the algebraic sum of the moment of these forces about each of B and M is vanished , then $F - K =$ .... newton.

(a) 3

(b) 6

(c) 12

(d) 9

8. A uniform ladder of weight 20 kg.wt. rests in a rough horizontal ground with one of its ends and the other end on a smooth vertical wall. The ladder is being kept in a vertical plane and inclined at 60° to the horizontal. If it is known that the coefficient of friction between the ladder and the ground equals $\frac{1}{2\sqrt{3}}$ Prove that the maximum distance a girl of weight 60 kg.wt. can ascend up the ladder is equal to half the length of the ladder.

9. ABCD is a rectangle in which AB = 8 cm. , BC = 12 cm. , E , F are the midpoints of $\overline{BC}$ and $\overline{AD}$ respectively , forces of magnitudes 24 , 36 , 30 , 18 newton act at $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CF}$, $\overrightarrow{FA}$ respectively find the two forces acting at $\overrightarrow{EA}$, $\overrightarrow{FC}$ to make the system in equilibrium.

(10) If a body of weight (w) is placed on a rough inclined plane to the horizontal at an angle of measure θ , a force of magnitude (w) acted on the body in the line of the greatest slope upward to make the body about to move , then $\mu_s + \tan\theta =$ ....

(a) sec θ

(b) csc θ

(c) sin θ

(d) cos θ

(11) A uniform thin lamina ABCD in the shape of isosceles trapeziume in which $\overline{AB} // \overline{DC}$, AB = 2 BC = 20 cm. , CD = 8 cm. determine the centre of gravity of the lamina. , then if the lamina is placed vertically to make its edge $\overline{CD}$ coincide to a horizontal table. Prove that the greatest weight can be suspended from the vertex A to make the lamina about to rotate = $\frac{2}{3}$ weight of the lamina.

12 ***Answer one of the following items :***

[a] A uniform thin lamina in the shape of isosceles triangle ABC in which AB = AC , $\overline{AD}$ is the height of the triangle where AD = 45 , a straight line is drawn parallel to $\overline{BC}$ and passing through the centre of gravity of the lamina to intersect $\overline{AB}$, $\overline{AC}$ at the points E , F respectively. Prove that centre of gravity of the quadrilateral EBCF lies on $\overline{AD}$ and at a distance 7 cm. from the point D

[b] A uniform rod $\overline{AC}$ of the length 15 ℓ is bent from point B where $\overline{AB}$ = 5 ℓ such that m ($\angle$ ABC) = 90° and the rod is suspended freely from end A. Prove that $\overline{BC}$ is inclined at an angle of θ where $\tan\theta = \frac{4}{5}$ to the horizontal.

13 $\overline{AB}$ is a uniform rod of length 120 cm. and weight 600 gm.wt is suspended in horizontal position by two vertical strings at the two points C , D on the rod where AC = 25 cm. , BD = 35 cm. and a weight F is suspended at the point E on it where AE = 30 cm. and the tension in the string at C is twice the tension in the string at D find F

14 ***Answer one of the following items :***

[a] A body of weight 38 newton is placed on a horizontal rough plane and the tangent of the friction force angle between the body and the plane is $\frac{1}{4}$, the body is pulled by a force make an angle with the horizontal of sine equals $\frac{3}{5}$ to make the body is about to move find the magnitude of the resultant reaction.

[b] $\overline{AB}$ is a rod of length 50 cm. and weight 20 newton acts in its midpoint , is moving in a vertical plane about a hinge at A. A cuple magnitude of its moment is 250 newton cm. acts in the rod in a vertical plane , find the reaction of the hinge and measure of the inclination angle with vertical in the equilibrium position.

15 If $\overrightarrow{F_1} // \overrightarrow{F_2}$, $\overrightarrow{F_1} = 3\hat{i} - 4\hat{j}$, $\|\overrightarrow{F_2}\| = 10$ force unit ; then $\overrightarrow{F_2}$ can be ....

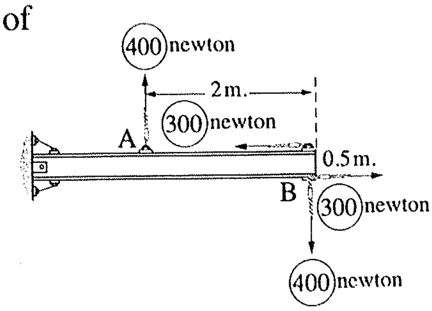
(a) $-3\hat{i} + 4\hat{j}$

(b) $6\hat{i} + 8\hat{j}$

(c) $-6\hat{i} + 8\hat{j}$

(d) $-6\hat{i} - 8\hat{j}$

16 The magnitude of the moment of the couple of the system of forces shown in the opposite figure in newton. m. equals ....



(a) 150

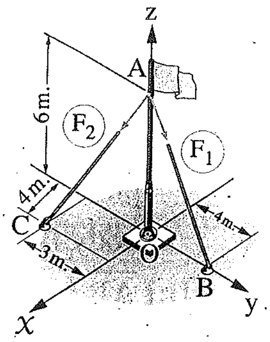
(b) 650

(c) 800

(d) 950

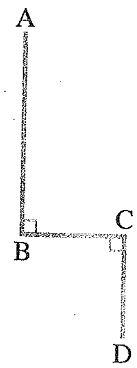
17 The force $F_1 = 6\sqrt{13}$ newton and $F_2 = \sqrt{61}$ newton in the direction of $\overrightarrow{AB}$ and $\overrightarrow{AC}$ as shown in the opposite figure.

Find the sum of the moments of the forces about point O



18 **In the opposite figure :**

ABCD is a wire of length 32 cm. in which AB = 2 BC = 2 CD = 16 cm. , then the distance between the centre of gravity of the wire and both $\overleftrightarrow{BC}$ and $\overleftrightarrow{BA}$ respectively is ....



(a) (3 , 3)

(b) (4 , 4)

(c) (3 , 5)

(d) (4 , 8)

Model 9

Answer the following questions :

1 A body of weight 12 newton is placed on a rough plane inclined to the horizontal by an angle of measure 60° if the coefficient of the static friction between the body and the plane $= \frac{\sqrt{3}}{9}$, then the body ....

(a) remained at rest on the plane.

(b) can't be at rest on the plane.

(c) is about to move upward the plane.

(d) is about to move downward the plane.

2 ABC is an equilateral triangle whose side length is 20 cm. , forces of magnitudes 1 , 2 , 3 newton acted in the directions $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{AC}$ respectively , then the sum of moments of the forces about the point of concurrence of its medians = ... N.cm.

(a) 20

(b) zero

(c) $10\sqrt{3}$

(d) $20\sqrt{3}$

3 Two parallel forces have the same direction , of magnitudes F_1 , F_2 Newton If $F_1 : F_2 = 1 : 2$ and their resultant = 15 newton , then $F_2 - F_1$ = ... newton.

(a) 5

(b) 7 – 5

(c) 10

(d) 15

4 If the magnitude of the moment of a couple is 30 newton. m and the length of its arm is 5 m. , then the magnitude of one of its forces in newton = ...

(a) 6

(b) 25

(c) 35

(d) 10

5. ABCD is right trapezeum at B , $\overline{AD} // \overline{BC}$, AB = 9 cm. , AD = 12 cm. , BC = 24 cm. , the point E is the midpoint of $\overline{BC}$, forces of magnitudes 27 , 72 , 45 , 36 newton acted on $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ respectively , find magnitude of each of the two forces have to be act on $\overleftrightarrow{EA}$ and $\overleftrightarrow{DC}$ to make the system in equilibrium.

6. The two parallel forces $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$ and $\overrightarrow{F_2}$ are acting at A (1 , 3) and B (4 , 9) respectively , if their resultant is acting at C (3 , 7) , then $\overrightarrow{F_2}$ = ...

(a) $\hat{i} - \frac{3}{2}\hat{j}$

(b) $-4\hat{i} + 6\hat{j}$

(c) $4\hat{i} - 6\hat{j}$

(d) $4\hat{i} + 6\hat{j}$

7. If the coefficient of static friction between a body and a plane is 2 sin 30° , then the measure of the angle of friction = ...

(a) 30°

(b) 45°

(c) 60°

(d) 75°

8 If the line of action of $\vec{F} \parallel \overline{AB}$, $\overrightarrow{M_A} = 15\,\hat{k}$

, then $\overrightarrow{M_B} = \dots\dots\dots$

ⓐ $\vec{0}$

ⓑ $-5\,\hat{k}$

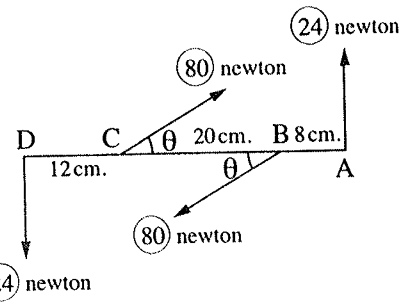
ⓒ $15\,\hat{k}$

ⓓ $30\,\hat{k}$

9 $\overline{AB}$ is a non uniform rod of length 70 cm. rests at its end B on a horizontal ground and at its end A on a vertical wall. If the coefficients of the static friction between the rod and each of the ground and wall are $\frac{1}{2}$, $\frac{1}{3}$ respectively , the rod was about to slide when the angle of its inclination with measure 45° , find the distance between the end B and the centre of gravity knowing that the rod lies in a horizontal plane perpendicular to the intersection line between wall and ground.

10 If $\overline{AD}$ is an equilibrium rod under the effect of the shown system of forces in the opposite figure and if AB = 8 cm. , BC = 20 cm. , CD = 12 cm.

, then $\sin\theta = \dots\dots\dots$



ⓐ 0.4

ⓑ 0.5

ⓒ 0.6

ⓓ 0.8

11 ***Answer one of the following items :***

[a] $\overline{AB}$ is non uniform rod of length 70 cm. and weight 4.5 kg.wt. rests in horizontal position on two smooth supports at C , D where AC = 12 cm. , BD = 14 cm. It found that if a weight 6 kg.wt. suspended from the end A , then the rod is about to rotate. Find the distance between the centre of gravity and the point C

[b] A body of weight 20 newton is placed on horizontal rough plane , the coefficient of the static friction between the body and the plane = $\frac{1}{4}$ find the force that inclined to the horizontal by an angle of measure 30° to make the body about to move.

12 A force $\overrightarrow{F}$ act at the point A (– 3 , 2) , if its moment with respect to each of B (3 , 1) , C (– 1 , 4) equals $28\,\hat{k}$, then $\overrightarrow{F}$ = ...

(a) $8\,\hat{i} + 6\,\hat{j}$

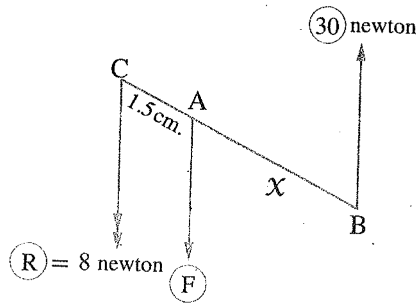
(b) $-8\,\hat{i} + 6\,\hat{j}$

(c) $-8\,\hat{i} - 6\,\hat{j}$

(d) $8\,\hat{i} - 6\,\hat{j}$

13 **In the opposite figure :**

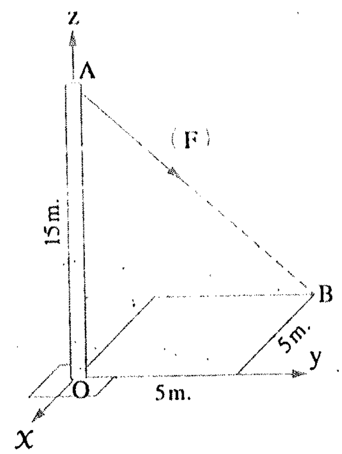
If 30 , F newton are the magnitudes of two parallel forces their resultant is R , then find the value of x



(14) A uniform rod of length 140 cm. and weight 6 kg.wt. act on its midpoint can be rotate easily about a horizontal fixed nail passing through a small hole in the rod at the point C at a distance 35 cm. from the end B if the rod rests with its end A on a smooth horizontal table. The end B is pulled horizontaly by a robe to make the reaction of the table equals to the weight of the rod , find magnitude and direction of the reaction of the nail knowing that the angle of inclination of the rod to the horizontal is of measure 30°

(15) In the opposite figure :

Find the moment of the force $F = 15\sqrt{11}$ newton about O



16 A regular thin lamina in the shape of an equilateral triangle is suspended by a string from one of its edges to divide it by the ratio 1 : 2 , then the measure of the angle of inclination of this edge to the vertical equals ....

(a) 22.5°

(b) 30°

(c) 45°

(d) 60°

17 Two bodies , their weights are 2 w , 3 w connected by light string coincides with the line of greatest slope of an inclined rough plane the coefficients of static frictions between them and the plane are $\frac{1}{4}$ and $\frac{1}{6}$ respectively , if θ is the measure of the angle between the plane and the horizontal plane is increased gradually , which of the two bodies should be placed below the other to make the two bodies move together keeping the string taut, state the reason , then prove that $\tan \theta = \frac{1}{5}$ when the two bodies were about to slide.

18 ***Answer one of the following items :***

[a] A uniform wire of length 120 cm. and mass 600 gm. is bent to form a Δ ABC which is right angled at B where AB = 30 cm. If a mass m is fixed at the vertex A , then the wire is suspended freely at the vertex B to be equilibrium when $\overline{AC}$ is horizontally , then find m

[b] Two bodies of masses 6 kg. , 12 kg. and distance between them 90 cm. Find the centre of gravity of the two bodies with respect to the body 6 kg.

Model 10

Answer the following questions :

1 If a body is placed on a rough inclined plane and it became about sliding on the plane , then the tangent of the angle of friction equals each of the following except ....

(a) the friction coefficient.

(b) the ratio between the magnitude of the normal reaction and the magnitude of the resultant reaction.

(c) the tangent of the inclination angle of the plane on the horizontal.

(d) the ratio between the limiting friction and the magnitude of the normal reaction.

2 If the line of action of $\vec{F} = \hat{i} + \hat{j}$ bisects $\overline{AB}$ where A = (3 , – 1) , and D = (1 , 4) is the midpoint of $\overline{AB}$ then $\overrightarrow{M_B}$ = ... $\hat{k}$

(a) – 7

(b) 7

(c) 3

(d) – 3.5

3 F_1 , F_2 are two forces where the magnitude of the first force = 4 kg.wt. and the magnitude of their resultant R = 6 kg.wt. the distance between F_1 and R = 8 cm. , then if F_1 and R act in the same direction , then the distance between F_1 and F_2 equals ....

(a) 12 cm.

(b) 16 cm.

(c) 20 cm.

(d) 24 cm.

4 The important and sufficient condition for the equilibrium of a set of forces is ....

(a) the vector sum of the set of forces is vanished.

(b) the set of forces are meeting at one point.

(c) the forces are parallel.

(d) the vector sum of the set of forces vanishes and the vector moment of the set of forces about any point vanishes.

5 ***Answer one of the following items :***

[a] ABCDEF is a regular hexagon of side length 8 cm. Forces of magnitudes 3 , 8 , 3 , 8 kg.wt. act on $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{DE}$, $\overrightarrow{FE}$ respectively.

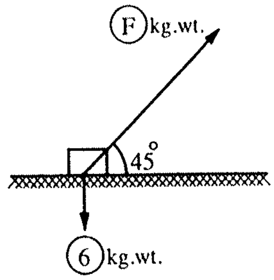
Prove that the set of forces is equivalent to a couple and find the norm of its moment

[b] Three forces of magnitudes 10.5 , 12 , 19.5 newton are completely represented by the directed line segments $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ respectively of the triangle ABC in which AC = 13 cm. Find the norm of moment of the couple which is equivalent to the three forces.

6 **In the opposite figure :**

A body of weight 6 kg.wt. is placed on a horizontal rough plane , the coefficient of friction between the body and the plane is $\frac{1}{2}$ A force of magnitude F kg.wt. acts on the body where it inclines with the plane by an angle of measure 45°

If the body is about to move , then F = ... kg.wt.



(a) $2\sqrt{2}$

(b) $3\sqrt{2}$

(c) $\frac{3}{\sqrt{2}}$

(d) $\frac{3\sqrt{2}}{2}$

7. Two parallel forces of magnitudes 5 , 12 newton act in opposite directions , then the magnitude of their resultant in newton equals ....

(a) 7

(b) 13

(c) 17

(d) 60

8. A uniform ladder of weight 40 kg.wt. rests at one of its ends on a vertical smooth wall , and the other end rests on a horizontal rough plane laying in the vertical plane perpendicular to the wall , such that it inclines at the horizon with an angle of measure 45° If a boy of weight equal that of the ladder ascended the ladder such that the ladder is about to slide when the boy covered $\frac{3}{4}$ the length of the ladder find the coefficient of friction between the ground and the ladder. If the boy wanted to complete his ascending to the ladder , find the least horizontal force acting on the lower end of the ladder for the boy can do that task.

9. If the force $\overrightarrow{F} = 15\hat{i} - 25\hat{j} + 40\hat{k}$ acts at the point (– 3 , – 3 , 2), then the component of the moment of the force $\overrightarrow{F}$ about y-axis = ... moment unit.

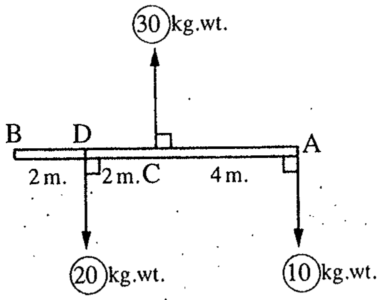
(a) 100

(b) 150

(c) 250

(d) 400

10 The algebraic measure of the moment of the couple of the set of forces shown in the opposite figure in kg.wt.m. unit equals ....



(a) – 80

(b) zero

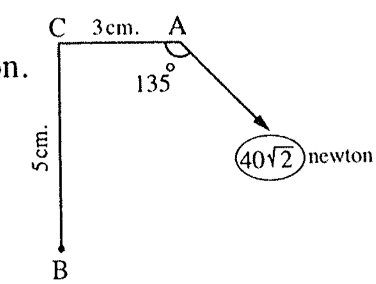
(c) 60

(d) 80

11 $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two parallel forces act in opposite in directions at the two points A and B respectively where $F_1 > F_2$ If the norm of its resultant equals 90 kg.wt. acting on $C \in \overleftrightarrow{BA}$ where AB = 36 cm. , AC = 16 cm. Find the magnitude of $\overrightarrow{F_1}$

12 **In the opposite figure :**

The magnitude of the moment of the force $F = 40\sqrt{2}$ newton. about the point B equals .... newton.cm.



(a) 320

(b) $200\sqrt{2}$

(c) $120\sqrt{2}$

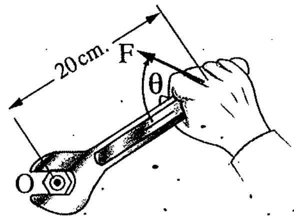
(d) $80\sqrt{16}$

13. A body of weight 4 kg.wt. is placed on a rough plane incline to the horizon with an angle of measure 30° , if the coefficient of friction between the body and the plane is $\frac{\sqrt{3}}{2}$, find the least and the greatest force acting on the body in the direction of the greatest slope such that it makes the body about to move.

14. $\overline{AB}$ is a rod of length 24 cm. and of weight 5 kg.wt. acting at its midpoint. It can rotate easily in a vertical plane about a fixed horizontal nail passing through a small hall in the rod at the point C which is far from B with distance 4 cm. If the rod rests with its end A on a smooth horizontal table. The end B is pulled horizontally with a string till the reaction of the table became equal to the weight of the rod. Find the reaction of the nail at this condition to make that the rod is kept in equilibrium at a position in which the rod inclines at the horizon with an angle of measure 30°

15 **In the opposite figure :**

If the moment required to make the nail rotate about O equals 400 newton.cm. Find the least value of the force F and the value of θ which rotates the nail.



16 **In the opposite figure :**

The centre of gravity of the following system with respect to the point A is ....

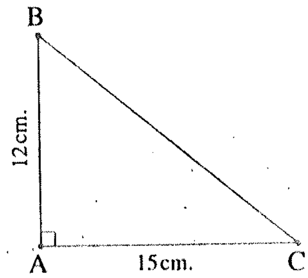
The mass	20 gm.	40 gm.	30 gm.
The position	at A	at B	at C

ⓐ (5 , 4)

ⓑ $\left(5 , \frac{16}{3}\right)$

ⓒ $\left(\frac{7}{3} , \frac{10}{3}\right)$

ⓓ $\left(\frac{7}{2} , 6\right)$



17 $\overline{AB}$ is a uniform rod of length 50 cm. , supporting horizontally on two supporters C and D at a distance 10 , 40 cm. from the end A. If a weight = 60 newton is susbended at A , then the rod becomes about rotating about C Find the weight of the rod.

18 ***Answer one of the following items :***

[a] A thin uniform wire in thickness and density , of length 40 cm. It is bent in the shape of the trapezium ABCD in which AB = 16 cm. , CD = 8 cm. , DA = 6 cm. m (∠ DAB) = m (∠ CDA) = 90° Find the distance between the centre of gravity of this wire and each of the two sides $\overline{AB}$ and $\overline{AD}$

[b] ABC is a triangular lamina of an equilateral triangle , of mass 3 kg. M is its centre of gravity masses of magnitudes 2 , 2 , 11 kg. are placed at the vertices A , B and C respectively , prove that the centre of gravity of this system lies at the midpoint of $\overline{MC}$

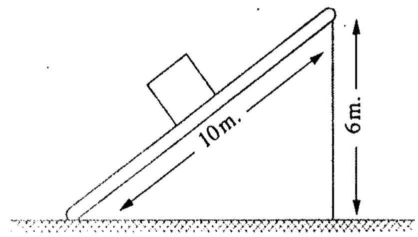
Model 11

Answer the following questions :

1 In the opposite figure :

The body is about to slide downwards the plane

, then the static coefficient of friction = ...



(a) $\frac{3}{5}$

(b) $\frac{4}{5}$

(c) $\frac{3}{4}$

(d) $\frac{4}{3}$

2 F_1 , F_2 are two forces , the magnitude of the first is 4 kg.wt. and the magnitude of their resultant R equals 6 kg.wt. , the distance between F_1 and R equals 8 cm. If F_1 and R act in two opposite directions , then the distance betwen F_1 and F_2 equals ....

(a) 3.2 cm.

(b) 4.8 cm.

(c) 9.6 cm.

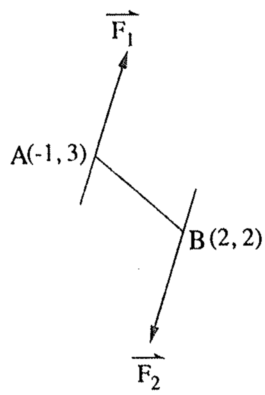
(d) 12.6 cm.

3 In the opposite figure :

If $\overrightarrow{F_1} = 2\hat{i} + 5\hat{j}$, $\overrightarrow{F_2} = -2\hat{i} - 5\hat{j}$

act at the two points A and B

, then the moment of the couple equals ....



(a) $-17\hat{k}$

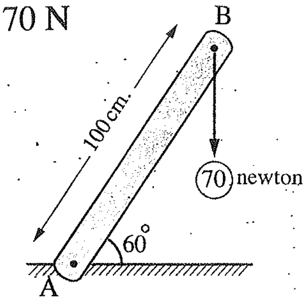
(b) $-13\hat{k}$

(c) $13\hat{k}$

(d) $17\hat{k}$

4 **In the opposite figure :**

A rod $\overline{AB}$ is fixed at a hinge at A , a vertical force of magnitude 70 N acts at the end B downwards , then the magnitude of the moment of this force about the point A equals .... N.m.



(a) 35

(b) $35\sqrt{3}$

(c) 70

(d) $70\sqrt{3}$

5 ***Answer one of the following items :***

[a] Two parallel forces , their resultant = 12 gm.wt. The magnitude of one of these forces is 15 gm.wt. , and acts at a distance 10 cm. from the resultant. Find the distance between the two forces if the given force acts in the same direction of the resultant.

[b] If $\vec{F} = 3\hat{i} - 4\hat{j}$ acts at the point A (0 , 2) , If C = (2 , 3) , D = (– 2 , 1) , prove that the line of action of the force $\vec{F}$ bisects $\overline{CD}$

6 The magnitude of the horizontal force which makes a body of weight 15 kg.wt. placed on a horizontal rough plane about to move where the measure of the angle of friction between the body and the plane is 30° equals ....

(a) $\frac{1}{15}\sqrt{3}$

(b) $\frac{1}{5}\sqrt{3}$

(c) $5\sqrt{3}$

(d) $15\sqrt{3}$

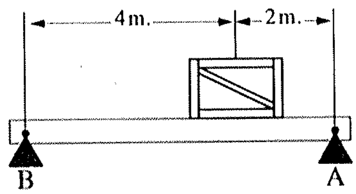
7 If the force $\overrightarrow{F} = 3\hat{i} - 4\hat{j} - 12\hat{k}$ acts at the point A (– 1 , 2 , 1)

Answer one of the following items :

[a] Find the moment of $\overrightarrow{F}$ about the origin point.

[b] Find the moment of the force $\overrightarrow{F}$ with respect to the point B (3 , 4 , 0)

8 The opposite figure represents a box of weight 60 newton placed on a wooden board (neglected weight) , then the pressure acting on the supporter A in newton equals ....



(a) 10

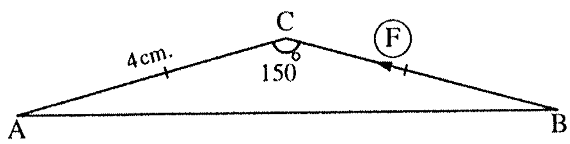
(b) 20

(c) 25

(d) 40

9 **In the opposite figure :**

Δ ABC is an isosceles triangle in which BC = AC = 4 cm. , m ($\angle$ C) = 150°



If the force $\overrightarrow{F}$ acts along $\overrightarrow{BC}$ where F = 15 newton , then the algebraic measure of the moment of the force $\overrightarrow{F}$ about A equals .... newton.cm.

(a) 60

(b) 30

(c) 120

(d) 15

10. ABCD is a rhombus of side length 12 cm. , m (∠ A) = 60° forces of magnitudes 50 , 80 , 50 , 80 gm.wt. act along $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$, $\overrightarrow{DA}$ respectively , prove that the system of forces is equivalent to a couple and find the magnitude of its moment.

11. If the moment of the force $\overrightarrow{F} = 3\,\hat{i} - \hat{j}$ about a point is $21\,\hat{i} + 7\,\hat{j}$, then the length of the perpendicular drawn from this point to the line of action of the force in length unit equals ....

(a) $\frac{1}{7}$

(b) $\frac{1}{7}\sqrt{10}$

(c) 7

(d) $10\sqrt{7}$

12 A thin wire of uniform thickness and density , it is bent in the form of a right-angled triangle at B in which AB = 3 cm. , BC = 4 cm. , then the distance between the centre of gravity of the wire and each of $\overline{BA}$, $\overline{BC}$ is ....

(a) (1.5 , 1)

(b) (2 , 1.5)

(c) $\left(\frac{8}{7} , \frac{9}{14}\right)$

(d) $\left(\frac{12}{7} , \frac{11}{14}\right)$

13 A uniform rod of weight (w) , it is connected at one of its ends with a hinge , and the other end is attached with a string fastened at a point in the same horizontal plane passing through the hinge such that the measure of the angle of inclination between each of the rod and the string with the horizon equals θ , prove that the reaction of the hinge = $\frac{1}{4}$ w$\sqrt{8 + \csc^2\theta}$

14 A body of weight $\sqrt{57}$ kg.wt. is placed on a rough horizontal plane. Two forces of magnitudes 2 , 3 kg.wt. act on the body such that they include an angle of measure 60° between them , and the two forces are horizontal and laying in the same horizontal plane of the body. If the body became about to move , find the static coefficient of friction between the body and the plane also find the measure of the angle of friction.

15 $\overline{AB}$ is a non-uniform rod of length 24 cm. and of weight 10 newton. it rests in horizontal state on two supporters at C and D where AC = BD = 5 cm. A weight of 20 newton is hanged at A , then the rod became about rotation about C Find the distance of the point of action of the rod's weight from A

16 **In the opposite figure :**

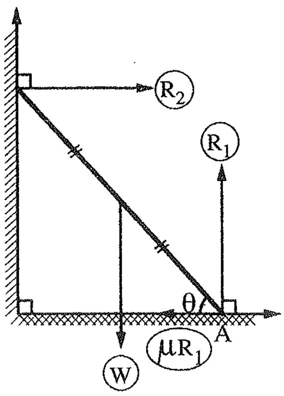
If λ is the measure of the angle of friction between the rod and the ground , then $\tan \theta . \tan \lambda =$ ....

(a) 3

(b) 2

(c) 1

(d) $\frac{1}{2}$



17 ABC is a thin lamina in the shape of a right-angled triangle at B , its weight = 6 newton act at the point of meeting of its medians , if AB = 12 cm. , BC = 15 cm.
It is suspended at a nail passing through a small hall at A such that its plane is vertical. A couple act on its plane to make it at rest when $\overline{AB}$ is vertical. Find the magnitude of the moment of the couple.

18 ABCD is a thin uniform lamina in the shape of a square of side length 48 cm. and its mass = 40 gm. L and M are the two midpoints of $\overline{AB}$ and $\overline{AD}$ respectively the triangle ALM is cut , then two masses each of them equal the mass of Δ ALM are fixed at the two points C and D , and a mass equals twice the mass of the separated triangle is fixed at B The system is suspended freely at the point C find the tangent of inclination of $\overline{BC}$ with vertical in the equilibrium position.

Model 12

Answer the following questions :

1. The force $\overrightarrow{F} = 3\hat{i} + \hat{j}$ acts at the point A (2 , – 1) , then the vector moment of this force with respect to the point B (1 , 2) equals ....

   (a) $7\hat{k}$
   (b) $-10\hat{k}$
   (c) $10\hat{k}$
   (d) $-7\hat{k}$

2. 2 F and 3 F are two parallel forces act in the same direction , their resultant = 35 newton , then the magnitude of the smaller force in newton equals ....

   (a) 7
   (b) 10
   (c) 14
   (d) 21

3. ABCD is a parallelogram in which AB = 6 cm. , BC = 8 cm. , m (∠ A) = 60° , forces of magnitudes 8 , 10 , 8 , 10 newton act along $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$ respectively. Find the magnitude of the moment of the couple which is equivalent to this system.

4. ABCD is a right trapezium at B , $\overline{AD} // \overline{BC}$, AB = 8 cm. , BC = 15 cm. , AD = 9 cm. Forces of magnitudes F , 44 , 68 gm.wt. act along $\overrightarrow{DA}$, $\overrightarrow{DC}$, $\overrightarrow{AC}$ respectively. If the line of action of the resultant of this system passes through the point B , then the value of F equals .... gm.wt.

(a) 114
(b) 126
(c) 156
(d) 184

5. The distance between the centre of gravity of two bodies whose masses are 3 and 5 kg. and the distance between them = 8 metre from the first body in metre equals ....

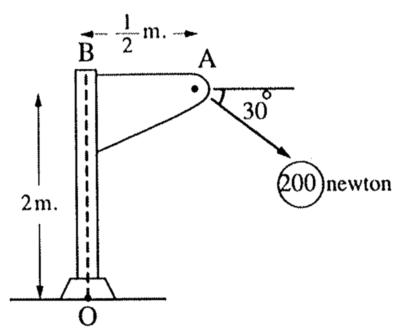
(a) 2
(b) 3
(c) 4
(d) 5

6. The least force needed to make a body of weight (w) about to move on a horizontal rough plane whose measure of its angle of friction is λ equals ....

(a) w sin λ
(b) w cos λ
(c) w tan λ
(d) w csc λ

7. **In the opposite figure :**

The algebriac measure of moment of the force whose magnitude = 200 newton. with respect to the point O equals .... N.metre.



(a) $-200\sqrt{3}$
(b) -50
(c) $-250\sqrt{3}$
(d) $-200\sqrt{3}-50$

8 Two parallel forces of magnitudes 15 and F newton. If the magnitude of their resultant = 25 newton and given that the force and the resultant are in two opposite directions , then the value of the force F in newton = ...

(a) 10

(b) 20

(c) 30

(d) 40

9 $\overline{AB}$ is a uniform rod of length 260 cm. and of weight 43 newton rests at its end A on a vertical wall and at its end B on a horizontal ground. The two coefficients of friction between the rod and each of the wall and the ground equal $\frac{1}{4}$ and $\frac{1}{2}$ respectively. The end B is at a distance 100 cm. from the wall. Find the magnitude of the horizontal force which acts at B to make the rod about to move towards the wall.

10 If the two forces $\overrightarrow{F_1} = 4\hat{i} + a\hat{j} + 5\hat{k}$, $\overrightarrow{F_2} = b\hat{i} - 7\hat{j} + c\hat{k}$ form a couple , then a + b + c = ...

(a) – 16

(b) – 2

(c) 2

(d) 16

11 The two forces $\overrightarrow{F_1} = 3\hat{i} - \hat{j}$, $\overrightarrow{F_2} = -9\hat{i} + 3\hat{j}$ act at the two points A (– 1 , 0) , B (1 , 2) respectively. Find the resultant of the two forces and determine the point of its action.

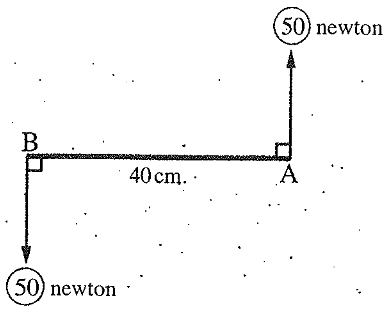
12 ***Answer one of the following items :***

[a] A body of weight 50 newton is placed on an inclined rough plane , which makes with the horizon on angle of measure θ. If the least force and greatest force which act parallel to the line of greatest slope to make the body in equilibrium on the plane are 10 and 40 newton respectively. Find the coefficient of friction and find also the measure of the angle of inclination of the plane to the horizon.

[b] If $\overrightarrow{F} = 3\hat{i} + 4\hat{j}$ acts at the point A (– 1 , 3) of a body find the length of the perpendicular drawn from O to the line of action of the force $\overrightarrow{F}$

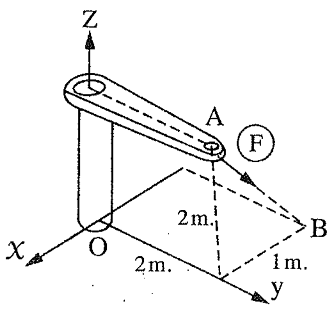
13 The algebraic measure of moment of the couple of the shown forces in the opposite figure in newton.cm. equals ....

(a) – 4000
(b) – 2000
(c) 2000
(d) 4000



14 **In the opposite figure :**

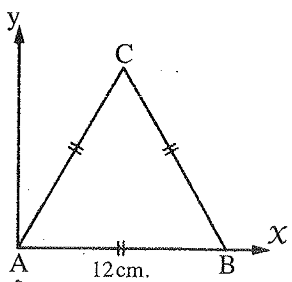
Calculate the moment of the force $F = 14\sqrt{5}$ newton about the point O



15 The centre of gravity of the following system is ....

The mass	4 gm.	5 gm.	3 gm.
The position	at A	at B	at C

(a) $\left(6\frac{1}{2}, \frac{3}{2}\sqrt{3}\right)$
(b) $\left(6\frac{1}{2}, 6\sqrt{3}\right)$
(c) $\left(6\frac{1}{2}, \frac{5}{3}\sqrt{3}\right)$
(d) $\left(9, 3\sqrt{3}\right)$



16 A rod $\overline{AB}$ of length 60 cm. and weight 400 gm.wt. acting at its midpoint , rests on a support at a distance 20 cm. from A. It is kept in equilibrium horizontally by a vertical light string connecting at B Find the magnitude of each of the tension in the string and the reaction of the support , then find the magnitude of the weight needed to be hanged at the end A to make the tension in the string about to vanish.

17 ABCD is a thin uniform lamina in the shape of a square of side length 20 cm. and of weight 200 gm.wt. acting at its geometrical centre. It is suspended by a nail passing through a small hole near by A such that its plane is vertical. A couple with moment of magnitude 1000 gm.wt.cm. acts on the lamina in a perpendicular direction to its plane to be in equilibrium. Find the measure of the angle of inclination of $\overline{AC}$ to the vertical.

18 ***Answer one of the following items :***

[a] $\overline{AC}$ is a uniform rod of length 2ℓ. It is bent at its midpoint then it is suspended from the end A freely. If $\overline{BC}$ is horizontal in equilibrium position , prove that : $\cos(\angle ABC) = \frac{1}{3}$

[b] A thin uniform lamina in the shape of the square ABCD of side length 48 cm. M is the point of intersecting of its diagonals , then Δ CMD is cut to be fixed on the triangle CMB such that $\overline{MD}$ coincides $\overline{MB}$, find the distance between the centre of gravity of the lamina and each of $\overline{BA}$, $\overline{BC}$

Model 13

Answer the following questions :

1 If the limiting force of friction = 30 newton , and the magnitude of the resultant reaction = 50 newton , then the static coefficient of friction equals ....

(a) $\frac{9}{16}$

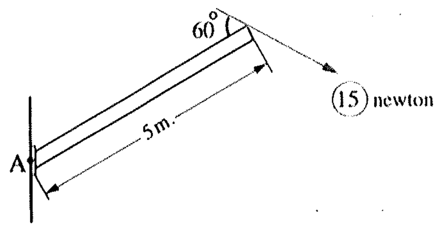
(b) $\frac{3}{4}$

(c) $\frac{4}{5}$

(d) $\frac{4}{3}$

2 **In the opposite figure :**

The algebraic measure of moment of the force whose magnitude = 15 newton about the point A equals .... newton.m.



(a) 75

(b) $\frac{75\sqrt{3}}{2}$

(c) $\frac{75}{2}$

(d) $75\sqrt{3}$

3 If $\overrightarrow{F_1} // \overrightarrow{F_2}$ and in the same direction where $F_1 = 50$ gm.wt. , $F_2 = 60$ gm.wt. , the distance between them = 44 cm. , then the distance between $\overrightarrow{R}$ and $\overrightarrow{F_1}$ = ... cm.

(a) 16

(b) 18

(c) 20

(d) 24

4 If $\overrightarrow{M_1}$ and $\overrightarrow{M_2}$ are two equilibrium couples and $\overrightarrow{M_1} = 20\hat{k}$, then $\overrightarrow{M_1} - \overrightarrow{M_2}$ = ...

(a) $40\hat{k}$

(b) $\vec{0}$

(c) 0

(d) $-40\hat{k}$

5 A uniform rod of weight (w) rests at one of its ends on a vertical rough wall and the other end on a rough horizontal ground , the coefficient of friction between the rod and the wall = $\frac{1}{4}$ and the coefficient of friction between the rod and the ground = $\frac{1}{3}$, if the rod became at rest in a vertical plane perpendicular to the wall. Find the tangent of the angle of inclination of the rod with the vertical when the rod is about to slide.

6 Three equal forces in magnitude are parallel and act at the vertices of the triangle ABC in the same direction , then their resultant acts at ....

(a) the centre of the circumcircle of the triangle.

(b) the point of intersection of the medians of the triangle.

(c) the point of intersection of the altitudes of the triangle.

(d) the point of intersection of the bisectors of the internal angles of the triangle.

7 The centre of gravity of two bodies of masses 3 kg. and 6 kg. and the distance between them equals 15 cm. is at a distance from the first body equals .... cm.

(a) 4

(b) 5

(c) 7.5

(d) 10

8 *Answer one of the following items :*

[a] If the force $\vec{F} = 2\hat{i} + b\hat{j} + \hat{k}$ acts at the point A (− 1 , 3 , − 2) and the component of the moment of $\vec{F}$ about X-axis equals − 3 moment unit. find the value of b then find the length of the perpendicular drawn from the origin point to the line of action of this force.

[b] If $\vec{F} = 3\hat{i} - 4\hat{j}$ acts at the point A (0 , 2) and C (2 , 3) , E (5 , − 1) , prove that the line of action of the force $\vec{F}$ is parallel to $\overleftrightarrow{CE}$

9 ABC is an equilateral triangle of side length 12 cm. A force of magnitude 20 newton acts along $\overrightarrow{BC}$, then the magnitude of the moment of this force with respect to the point A is .... newton.cm.

(a) $120\sqrt{3}$

(b) 240

(c) 120

(d) $12\sqrt{3}$

10 If A , B and C are three non-collinear points where a set of forces act in its plane such that it forms a couple and it is found that $2 M_A + 3 M_B + 5 M_C = 120$ newton.cm. , then $3 M_A - M_C =$ .... newton.cm.

(a) 12

(b) 24

(c) 36

(d) zero

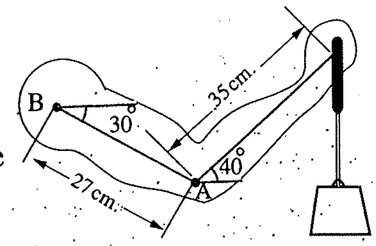
11 Two parallel forces of magnitudes 3 and 4 kg.wt. act at the two points A and B respectively in the same direction. If the first force is displaced such that it is parallel to itself , a distance ℓ on the ray $\overrightarrow{BA}$, prove that resultant of these two forces displaced a distance $= \frac{3}{7}\ell$ cm. in the same direction.

12 ***Answer one of the following items :***

[a] If $\overrightarrow{F_1} = 6\hat{i} + 3\hat{j}$ acts at the point A (1 , – 3) , and $\overrightarrow{F_2} = -\hat{i} - 4\hat{j}$ acts at the point B (2 , 5) , and $\overrightarrow{F_3} = \hat{i} - 5\hat{j}$ acts at the point C (4 , – 1) , prove that this system form a couple and find the magnitude of its moment.

[b] ABCDEF is a regular hexagon of side length 15 cm. forces of magnitudes 40 , 50 , 30 , 40 , 50 , 30 newton act along $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{DE}$, $\overrightarrow{FE}$, $\overrightarrow{FA}$ respectively determine the moment of the resultant couple.

13 The opposite figure represents a person carrying with his hand a weight.



If the moment of the weight about A = 80 newton. metre.
The moment of the weight about B ≃ .... newton.metre

ⓐ 140

ⓑ 150

ⓒ 180

ⓓ 210

14 If three equal masses are placed at the vertices of Δ ABC where A (2 , 1) , B (3 , 4) , C (4 , 1) , then the centre of gravity of this system is ....

ⓐ (2 , 3)

ⓑ (3 , 2)

ⓒ (6 , 9)

ⓓ (9 , 6)

15 A body of weight 200 gm.wt. is placed on a rough plane inclining with the horizon with an angle whose sine = $\frac{1}{2}$, the coefficient of friction between the body and the plane = $\frac{\sqrt{3}}{4}$. A force whose magnitude 25 gm.wt. acts in the direction of the greatest line slope upwards. If the body is at rest find the force of friction and show if the body is about to move or not.

16 $\overline{AB}$ is a non-uniform rod of length 120 cm. rests in a horizontal situation on two supporters at C and D of the rod where AC = 30 cm. , BD = 40 cm. If a weight of magnitude 160 gm.wt. is suspended at A , then the rod will be about to rotate about C and if a weight of magnitude 500 gm.wt. is suspended at B without removing that at A , then the rod will be about to rotate about D. Find the weight of the rod.

17 ABCD is a thin lamina in the shape of a square of side length 50 cm. and of weight 300 gm.wt. acting at the centre of the square. The lamina is suspended at a small hole near by the vertex A by a horizontal nail such that its plane is vertical.
A couple of algebraic measure of moment = 7500 gm.wt.cm. act on the lamina.
Find the inclination of the diagonal $\overline{AC}$ with the vertical in the equilibrium position.

18 A thin uniform lamina in thickness and density in the form of a triangle ABC which is isosceles where AB = AC = 26 cm. , BC = 20 cm. $\overrightarrow{AD}$ is drawn to be perpendicular to $\overline{BC}$ which cuts it at D. If E is the midpoint of $\overline{AD}$. The Δ EBC is separated. Find the distance between the centre of gravity of the remained part from the point E

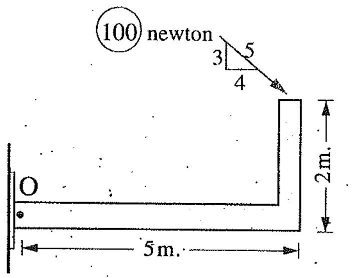
Model 14

Answer the following questions :

1 A body of weight 240 kg.wt. , placed on a horizontal rough plane. It is wanted to be pulled by a string making with the horizon an angle of measure 30° upwards. If the static coefficient friction equals 0.3 Find the magnitude of tension which is needed to make the body about to move to the nearest two decimals.

2 **In the opposite figure :**

The algebraic measure of the moment of the force whose magnitude = 100 newton about the point O equals .... newton.m.



(a) – 160

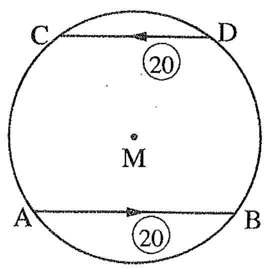
(b) – 460

(c) – 300

(d) 450

3 **In the opposite figure :**

Two forces each of them is of magnitude 20 kg.wt. , DC = 12 cm. , BA = 16 cm. , the radius length of the circle = 10 cm. , then the algebraic measure of the moment of the couple equals .... kg.wt. cm.



(a) 280

(b) 320

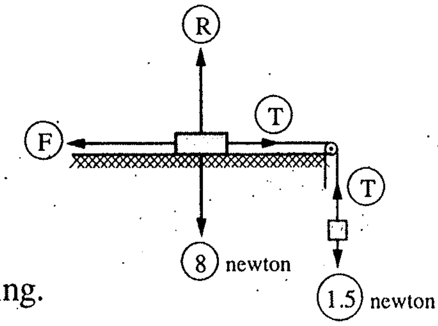
(c) 120

(d) 240

4 **In the opposite figure :**

If the coefficient of friction = $\frac{1}{4}$

, then ....



(a) the body is about to move.

(b) the body moves on the plane.

(c) the friction between the body and the plane is limiting.

(d) the friction between the body and the plane is not limiting.

5 If the force $\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$ acts at the point A (1 , − 1 , 4)

, then the length of the perpendicular drawn from B (2 , − 3 , 1)

on the line of action of the force = ... length unit.

(a) 3.73

(b) 13.96

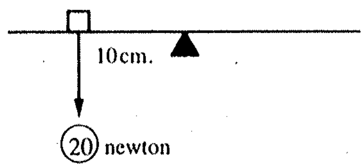
(c) 37.3

(d) 0.27

6 **In the opposite figure :**

A uniform rod rests on a supporter at its midpoint.

A body is placed on it as shown in the figure. Which of the following forces makes the rod in equilibrium ?



(a) A force of magnitude 10 newton upwards acts at a distance 20 cm. at the right side of the midpoint of the rod.

(b) A force of magnitude 10 newton downwards acts at a distance 20 cm. at the right side of the midpoint of the rod.

(c) A force of magnitude 30 newton downwards acts at a distance 5 cm. at the left side of the midpoint of the rod.

(d) A force of magnitude 30 newton upwards acts at a distance 5 cm. at the left side of the midpoint of the rod.

7. $\overline{AB}$ is a uniform rod of length 60 cm. and of weight 8 newton is connected with a hinge fixed on its end A at a vertical wall. A weight equals 6 newton is suspended from a point of the rod at a distance 40 cm. from the end A , then the rod became at rest horizontally by a light string attached from one of its terminals at B and the other terminal of the string is fixed at a point on the wall at a distance 80 cm. vertically up the point A
Find the tension in the string and the reaction of the hinge.

8. The centre of gravity of the system formed from $m_1 = 1$ at $(2 , 3)$, $m_2 = 2$ at $(-2 , 1)$, $m_3 = 3$ at $(0 , 1)$ is the point ....

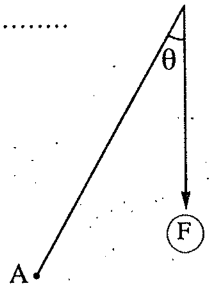
(a) $\left(-\frac{1}{3} , \frac{4}{3}\right)$
(b) $\left(\frac{7}{6} , \frac{4}{3}\right)$
(c) $\left(-\frac{1}{3} , \frac{2}{3}\right)$
(d) $(0 , 1)$

9. A body of weight 200 gm.wt. is placed on a rough inclined plane. A force F acts on it in the direction of the line of the greatest slope upwards. If the body is about to move downwards when F = 80 gm.wt. and the body will be about to move upwards when F = 120 gm.wt. , then the measure of the angle of inclination of the plane with the horizon equals ....

(a) $22\frac{1}{2}^\circ$
(b) 30°
(c) 45°
(d) 60°

10 **In the opposite figure :**

The greatest moment of the force F about the point A is when θ equals ....



(a) zero

(b) $\frac{\pi}{2}$

(c) π

(d) 2π

11 If $\vec{F} \,//\, \overrightarrow{AB}$, $C \in \overleftrightarrow{AB}$ and $\overrightarrow{M}_A + \overrightarrow{M}_B = 16\,\hat{k}$, then $\overrightarrow{M}_C =$ .... $\hat{k}$

(a) 16

(b) 8

(c) − 8

(d) 4

12 ***Answer one of the following items :***

[a] A , B , C , D lie on one horizontal straight line such that AB = 5 cm. , BC = 10 cm. , CD = 15 cm. Forces of magnitudes 4 , 6 , 2 , 8 kg.wt. act at the points A , B , C and D vertically downwards respectively. Find the distance between the point of action of the resultant from the point A

[b] Two parallel forces , the small of them is of magnitude 30 newton and acts at the end A from a light rod $\overline{AB}$ and the great force acts at the other end B , if the magnitude of their resultant is 10 newton and acts at a distance 90 cm. from the end B Find the length of the rod.

13 ABC is a lamina in the shape of an isosceles triangle in which AB = AC = 13 cm. , BC = 10 cm. It can rotate easily in a vertical plane about a hinge fixed at A If a couple of moment 200 gm.wt.cm. in magnitude acts on the lamina to be at rest when one of the two legs of the triangle is vertical. Find the weight of the lamina given that its acts at the point of intersecting of the medians of the triangle.

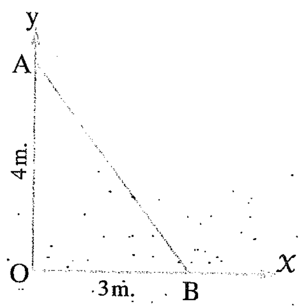
14 **In the opposite figure :**

The force $\overrightarrow{F}$ acts in the plane of the triangle AOB

If the moment of $\overrightarrow{F}$ with respect to the point O = 84 newton.metre.

and its moment with respect to the point A = – 100 newton. metre.

, and its moment with respect to the point B = zero Determine $\overrightarrow{F}$.



15 ***Answer one of the following items :***

[a] ABCD is a trapezium in which $\overline{AD} // \overline{BC}$, m (∠ ABC) = 90° , AB = 8 cm. , BC = 17 cm. , AD = 11 cm. Forces of magnitude 22 , 16 , 34 , 20 gm.wt. act along $\overrightarrow{DA}$, $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ respectively prove that the system is equivalent to a couple and find the magnitude of its moment.

[b] Five equal masses are placed at the points A , B , C , D , E where ABCD is a square , E is the point of intersection of its diagonals , and the length of its side is 12 cm. Determine the centre of gravity of this system. Then if the mass at B is separated , then determine the centre of gravity of the remained masses with respect to $\overrightarrow{AD}$ and $\overrightarrow{AB}$ as axes.

16 If the forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$, $\overrightarrow{F_3}$ act at the points (0 , 0) , (1 , 0) , (0 , 1) and they are equivalent to a couple where $\overrightarrow{F_1} = 3\hat{i} + 4\hat{j}$, $\overrightarrow{F_2} = -\hat{i} + \hat{j}$, then the magnitude of the couple equals ………… moment unit.

(a) 2

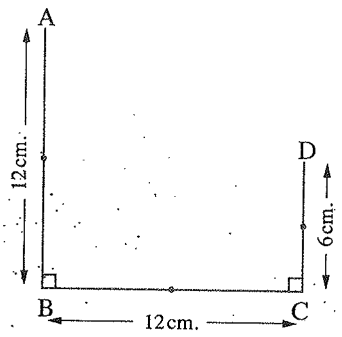
(b) 4

(c) – 2

(d) 3

17 **In the opposite figure :**

$\overline{AD}$ is a wire , of uniform thickness and density. It is bent at B and C , find the distance of the centre of gravity from each of $\overline{AB}$ and $\overline{CB}$, then find in the equilibrium position the measure of inclination of $\overline{AB}$ to the vertical if the wire is hanged freely from A



18 $\overline{AB}$ is a board of wood of length 20 metre and weight 50 kg.wt. acting at its midpoint. It is supported horizontally on two supporters one of them is at a distance 2 metre from A and the other is at a distance 5 metres from B. If a man of weight 70 kg.wt. walks on the board starting from A towards B Find the reaction of each supporter on the board when the man was at A , then find the maximum distance which the man can cover on the board without turning over to the board.

Model 15

Answer the following questions :

1 If the force $\vec{F} = \hat{i} - 2\hat{j}$ acts at the point A (2 , 3) , then the length of the perpendicular drawn from the point B (2 , 1) on the line of action of the force = ... length unit.

(a) $\frac{\sqrt{5}}{2}$

(b) $\frac{\sqrt{5}}{5}$

(c) $\frac{2\sqrt{5}}{5}$

(d) $\frac{\sqrt{2}}{5}$

2 Two parallel forces act in the same direction , their magnitudes are F and 3 F act at the two points A and B respectively where AB = 60 cm. , then the resultant act at the point C where AC = ... cm.

(a) 36

(b) 40

(c) 45

(d) 50

3 If the two forces $\vec{F_1} = 5\hat{i} + a\hat{j} + 3\hat{k}$, $\vec{F_2} = b\hat{i} - 9\hat{j} + c\hat{k}$ form a couple then a + b + c = ...

(a) – 1

(b) 1

(c) 8

(d) 17

4 If the angle of friction is λ , R is the normal reaction , then the resultant reaction $\grave{R}$ = ...

(a) $R\sqrt{1 + \tan\lambda}$

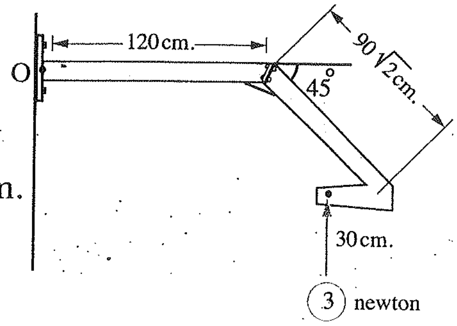
(b) $R \tan\lambda$

(c) $R\sqrt{1 + \sec\lambda}$

(d) $R \sec\lambda$

5 **In the opposite figure :**

The algebraic measure of moment of the force whose magnitude = 3 newton. with respect to the point O equals .... newton.cm.



(a) 630
(b) 720
(c) 450
(d) 540

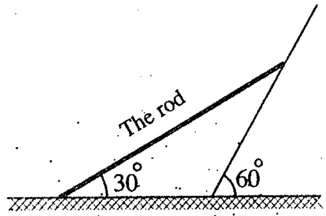
6 A body of weight 21 newton. is placed on a horizontal rough plane. Two horizontal forces of magnitudes 3 and 5 newton. act on the body where they include an angle on measure 60° between them , then the body became about to move , then the static coefficient of friction equals ....

(a) 3
(b) $\frac{1}{3}$
(c) 7
(d) $\frac{1}{7}$

7 A , B , C , D and E are collinear points , lie on one horizontal straight line such that AB = 2 BC = CD = 2 DE = 10 cm. forces of magnitudes 3 , 5 , 4 kg.wt. act vertically upwards at the points A , C , E and the forces of magnitudes 6 and 10 kg.wt. act vertically downwards at the two points B and D find the distance of the point of action of the resultant from the point E

8 In the opposite figure :

A uniform rod of weight 24 kg.wt. rests with one of its ends on a horizontal ground and with the other end on a smooth plane inclining with the horizon with an angle of measure 60°



If the rod is about to slide when it makes with the horizon an angle of measure 30°

Find the coefficient of friction between the rod and the ground also find the reaction of each of the plane and the ground.

9 ABC is a right-angled triangle at A where AB = AC = 5 cm. forces of magnitudes 40 , 40 , $40\sqrt{2}$ newton act along $\overrightarrow{BA}$, $\overrightarrow{AC}$, $\overrightarrow{CB}$ respectively. Find two forces to act at the points B and C perpendicular to $\overline{BC}$ in order that the system will be in equilibrium.

10 ***Answer one of the following items :***

[a] A uniform rod of length 64 cm. and weight 70 newton rests horizontally on two supports one of them is at a distance 8 cm. from one of the two ends and the other is at a distance 14 cm. from the other end. Find the weight that will be suspended from the other end so that the rod will be about over turning.

[b] A body of weight 14 kg.wt. is placed on a horizontal rough plane , when the body is pulled with a horizontal force of magnitude 7 kg.wt. then the body will be about to move. If a weight of magnitude 6 kg.wt. is placed on the body. Find the magnitude of the horizontal force which act on the body to make it and the weight about to move.

11 ABCDEF is a regular hexagon of side length 4 cm. Forces of magnitudes 1 , 3 , 5 , 2 , 4 and 6 newton. act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$, $\overrightarrow{DE}$, $\overrightarrow{EF}$, $\overrightarrow{AF}$ respectively , then the magnitude of the algebraic sum of the moments of these forces about the centre of the hexagon (M) equals ....

(a) $\sqrt{3}$

(b) $2\sqrt{3}$

(c) $4\sqrt{3}$

(d) $8\sqrt{3}$

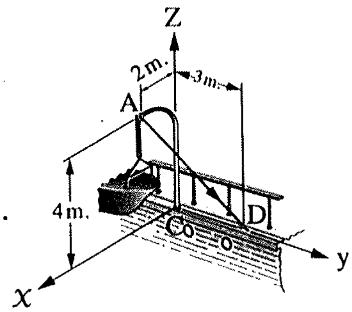
12 A body of weight 8 kg.wt. is placed on a horizontal rough plane then the plane is inclined gradually till the body became about to slide downwards the plane when the measure of the angle of inclination of the plane with the horizon is 30°. If the body is connected with a string , then it is pulled in a direction making an angle of measure 30° with the plane till the body became about to move upwards. Find the magnitude of the normal reaction.

13 **In the opposite figure :**

A rope fixed at the point D passes over a smooth pulley at A and a small boat is hanged at the other end of the rope.

If the tension in the rope $\overline{DA} = 10\sqrt{29}$ newton. in magnitude.

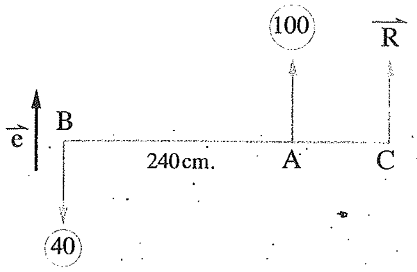
Find the moment of the tension in the rope about the point C



14 In the opposite figure :

$\overrightarrow{R}$ is the resultant of the two parallel forces 100 and 40

If AB = 240 cm. then AC = ...



ⓐ 100 cm.

ⓑ 120 cm.

ⓒ 160 cm.

ⓓ 200 cm.

15 ***Answer one of the following items :***

[a] A thin lamina is uniform in the thickness and density in the shape of square ABCD of side length 48 cm. M is the point of intersection of its diagonals. Δ CMD is cut to be fixed on Δ CMB such that $\overline{MD}$ coincides on $\overline{MB}$ Find the distance of the centre of gravity of the lamina from each of $\overline{BA}$ and $\overline{BC}$

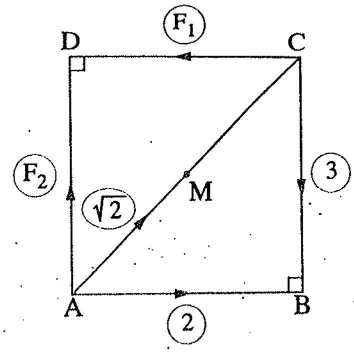
[b] Find the centre of gravity of the following system. $M_1 = 1$ kg. at the position A (2 , 3) , $M_2 = 2$ kg. at the position B (– 2 , 1) , $M_3 = 3$ kg. at the position C (0 , 1)

16 In the opposite figure :

ABCD is a square. The shown forces are measured in dyne.

If the set of forces are in equilibrium

, then $F_1 - F_2 =$ .... dyne.



(a) 3

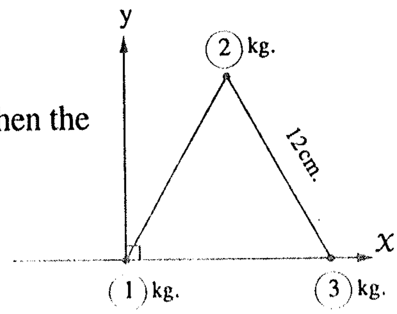
(b) 2

(c) 1

(d) – 1

17 The opposite figure represents an equilateral triangle of side length 12 cm.

Masses of magnitudes 1 , 2 , 3 kg. are placed at its vertices , then the centre of gravity of the system is ....



(a) $(4 , \sqrt{3})$

(b) $(\sqrt{3} , 4)$

(c) $(8 , 2\sqrt{3})$

(d) $(2\sqrt{3} , 4)$

18 ABC is a uniform lamina in thickness and density in the shape of an equilateral triangle of side length $18\sqrt{3}$ cm. and of weight 100 gm.wt. acting at the point of intersection of its medians. The lamina is hanged at a horizontal thin nail perpendicular to the plane of the lamina in a small hole near by the vertex A such that the plane of the lamina is vertical. If a couple act on the lamina in its plane to be in equilibrium when $\overline{AB}$ is horizontal. Find the magnitude of the moment of this couple.

Model 16

Answer the following questions :

1 If the static coefficient of friction between a body and a rough inclined plane $= \sqrt{3}$, then the measure of the inclination angle of this plane with the horizon when the body is about to slide under effect of its weight only = ...

(a) 30°

(b) 45°

(c) 60°

(d) 90°

2 The forces $\overrightarrow{F_1} = \hat{i} + \hat{j}$, $\overrightarrow{F_2} = \hat{i} - \hat{j}$, $\overrightarrow{F_3} = 2\hat{i} - 3\hat{j}$ act at the point A (1 , 1) , then the length of the normal drawn from the origin point to the line of action of the resultant of these forces = ... length unit.

(a) $\frac{5}{7}$

(b) 7

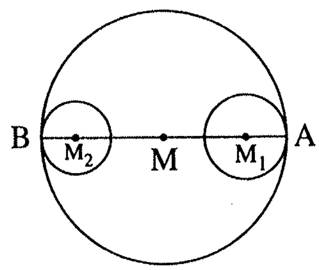
(c) 1.4

(d) 5

3 The resultant of two parallel forces = 40 newton and the magnitude of one these forces = 60 newton and its line of action is at a distance 24 cm. far from the line of action of the resultant. Find the distance between the two lines of action of the two parallel forces if the resultant and the given force act in the same direction.

4. ABCD is a rectangle in which AB = 12 cm. , BC = 8 cm. X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{CD}$. Forces of magnitudes 20 , 15 , F , 20 , 15 , F newton along $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{XC}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$, $\overrightarrow{YA}$ respectively. If the system is equivalent to a couple of magnitude of moment = 100 newton.cm. It acts in direction of ABCD , find the value of F

5. The opposite figure shows a circular disk , M is its centre. It is holed two circular holes of centres M_1 and M_2 and their radii lengths are 4 cm. and 3 cm. respectively , then the centre of gravity of the remained part lies on ....



(a) $\overline{MM_1}$

(b) $\overline{MM_2}$

(c) $\overline{AM_1}$

(d) $\overline{BM_2}$

6 ***Answer one of the following items :***

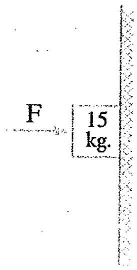
[a] If the force $\vec{F} = k\,\hat{i} + m\,\hat{j} - 2\,\hat{k}$ acts at the point A where its position vector with respect to the origin point is $\vec{r} = (3 , 1 , 1)$. If the two components of the moment of the force $\vec{F}$ about X-axis and y-axis are -1 ; -8 respectively.

Find the value of each of k and m

[b] A body of weight 12 newton is placed on horizontal rough plane. It is tied by a horizontal string which passes over a smooth pulley fixed at the edge of the table. A weight of magnitude 4 newton. is hanged vertically from the other end of the string. If the body is in equilibrium on the table , find the force of the friction. If the static coefficient of friction between the body and the table equals $\frac{1}{3}$. Is the body about to move ?

7 **In the opposite figure :**

The magnitude of the least horizontal force needed to make a body of mass 15 kg. in equilibrium on a rough vertical wall which the static coefficient of friction between the wall and the body $= \frac{1}{5}$ is .... kg.wt.



(a) 3

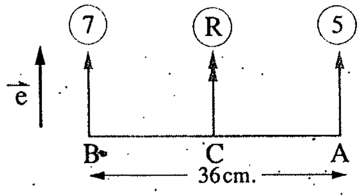
(b) 15

(c) 45

(d) 75

8 In the opposite figure :

If R is the resultant of the two parallel forces 5 , 7 kg.wt. , AB = 36 cm. , then the length of $\overline{AC}$ in cm. equals ....



(a) 15

(b) 18

(c) 21

(d) 24

9 ABC is a triangle in which AB = 10 cm . , BC = 16 cm. , m (∠ B) = 60°

Forces of magnitudes 2.5 , 4 , 3.5 kg.wt. act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ respectively , then the system is equivalent to a couple , the magnitude of its moment is .... kg.wt.cm.

(a) 20

(b) $20\sqrt{3}$

(c) 5

(d) $5\sqrt{3}$

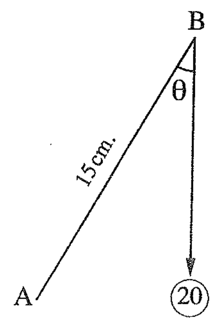
10 $\overline{AB}$ is a uniform rod of weight 12 newton and of length 60 cm. , it can rotate easily about a fixed nail passing through a hole in the rod , at a distance 10 cm. from B. It rests with its end A on a smooth horizontal surface. The end B is pulled by a horizontal force $\vec{F}$ till the pressure on the horizontal surface equal to the weight of the rod when the rod is inclined to the horizontal with an angle of measure 30° Find the magnitude of the force $\vec{F}$, also find the magnitude and direction of the reaction of the nail.

11 ***Answer one of the following items :***

[a] ABC is a triangular lamina in the shape of an equilateral triangle and its mass = 3 kg. M is its centre of gravity. Masses of magnitudes 2 , 2 , 11 kg. are placed at the vertices A , B and C respectively. Prove that the centre of gravity of the system lies at the midpoint of $\overline{MC}$

[b] ABCD is a rectangle in which AB = 8 cm. , BC = 12 cm. masses of magnitudes 2 , 4 , 5 , 7 gm. is fixed at the vertices A , D , B and C respectively. Also a mass = 12 gm. if fixed at the midpoint of $\overline{AD}$ Find the centre of gravity of system with respect to $\overrightarrow{CD}$, $\overrightarrow{CB}$

12 **In the opposite figure :**



The magnitude of the moment in newton.cm. of the force 20 newton about the point A ∈ ....

ⓐ [0 , 15]

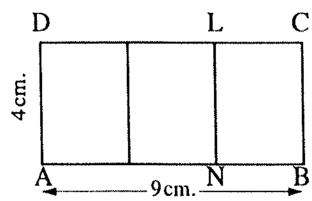
ⓑ [0 , 20]

ⓒ [0 , 30]

ⓓ [0 , 300]

13 An inclined rough plane , inclines to the horizontal with an angle whose tangent = $\frac{4}{3}$ a body of weight 10 newton. is placed on it. Find the minimum and maximum limits of the magnitude of the force which acts on the body in the direction of the line of the greatest slope of the plane upwards to make the body about to move given that the coefficient of friction between the body and the plane = $\frac{1}{2}$

14 The opposite figure shows a thin uniform rectangular lamina of dimensions 9 cm. and 4 cm. It is divided into 3 congruent rectangles. If the lamina is bent at $\overleftrightarrow{LN}$ till the part BCLN is touching the remaining of the lamina , then the distance of centre of gravity from $\overleftrightarrow{AD}$ equals .... cm.



(a) 3

(b) $3\frac{1}{2}$

(c) 4

(d) 4.2

15 If the moment vector of a force with respect to the point C $(2, 3)$ equals $6\hat{k}$ and its moment vector with respect to the point D $(0, -1)$ equals $-6\hat{k}$, then its moment vector with respect to the point .... $= \vec{0}$

(a) $(2, 2)$

(b) $(1, 1)$

(c) $(6, -6)$

(d) $(0, -3)$

16 If the magnitudes of two parallel forces acting in the same direction are $\frac{x}{y}$, $\frac{y}{x}$ newton. and their resultant = 2 newton. in magnitude, then ....

(a) $x = y$

(b) $x = 2y$

(c) $y = 2x$

(d) $x = y^2$

17 $\overline{AB}$ is a rod of length 120 cm. and of weight 80 gm.wt. acting at its midpoint. The rod rests horizontally on a support at B and it is kept in equilibrium by a light vertical string fixed at a point on the rod at distance 20 cm. from A, it carries a weight of magnitude 25 gm.wt. at a point at a distance 15 from A. Find each of the tension in the string and the pressure on the support. Find also the weight that should be hanged at A so that the rod becomes about separating from the support.

18 A uniform ladder rests at one of its ends on a vertical wall , the coefficient of friction between the wall and the ladder equals $\frac{1}{3}$ and it rests at the other end on the rough horizontal ground of the same roughness of the wall. If the ladder is in equilibrium in a vertical plane at a position in which the ladder inclines with the wall with an angle of tangent $\frac{6}{11}$ Prove that a man of weight equals 3 times the weight of the ladder can not ascend more than $\frac{7}{10}$ the length of the ladder without sliding the ladder.

Model 17

Answer the following questions :

1 If λ is the measure of the angle between the limiting friction and the resultant reaction , then the coefficient of friction equals ....

(a) $\tan \lambda$

(b) $\sin \lambda$

(c) $\cos \lambda$

(d) $\cot \lambda$

2 Two parallel forces of magnitudes 15 and F newton. If the magnitude of their resultant is 25 newton and the given force and the resultant act in the same direction , then the value of F in newton equals ....

(a) 10

(b) 20

(c) 30

(d) 40

3 If the mass 5 kg. is located at the point $(2 , -1)$, and the mass 7 kg. is located at the point $(1 , 2)$, then the centre of gravity of the two masses acts at the point ....

(a) $(17 , 9)$

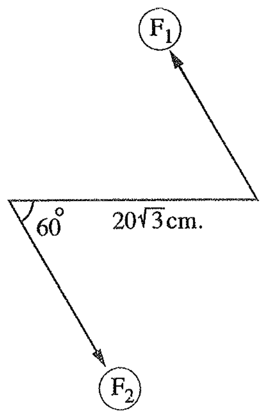
(b) $\left(\frac{17}{12} , \frac{3}{4}\right)$

(c) $(19 , 13)$

(d) $\left(\frac{19}{12} , \frac{1}{4}\right)$

4 In the opposite figure :

If $F_1 = 7$ newton and the two forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ form a couple , then the algebraic measure of the moment of the couple equals .... newton.cm.



(a) $140\sqrt{3}$

(b) $70\sqrt{3}$

(c) 210

(d) 105

5 A uniform rod $\overline{AB}$ of length ℓ is in equilibrium in a vertical plane when it rests at its end A on a smooth vertical wall and its end B rests on a horizontal rough ground , the coefficient of friction between it and the rod equals $\frac{1}{3}$ If the measure of the angle of inclination of the rod with the vertical is given by the relation $\tan \theta = \frac{1}{2}$. How far is the point on the rod from the end B at which a weight equals double the weight of the rod is suspended so that the rod is about to slide ?

6 Two parallel forces of magnitudes 70 newton and 30 newton , act in opposite directions and the distance between their lines of action is 50 cm. Find their resultant.

7 If the force $\vec{F} = 3\hat{i} + p\hat{j} + 4\hat{k}$ acts at the point A (1 , 0 , – 1) and the moment of the force $\vec{F}$ with respect to the point B (2 , – 1 , 3) equals $-4\hat{i} - 8\hat{j} - \hat{k}$, then p = ...

(a) – 2

(b) – 1

(c) 1

(d) 2

8 *Answer one of the following items :*

[a] Five equal masses are located at the vertices A , B , C , D and E of a square where E is the point of intersection of the two diagonals , the side length of the square = 12 cm. Determine the centre of gravity of this system. Then if the mass at B is removed , determine the centre of gravity of the remaining masses with respect to $\overrightarrow{AD}$, $\overrightarrow{AB}$

[b] Masses of magnitudes 10 , 20 , 10 , 30 , 10 , 40 kg. are fixed at the vertices A , B , C , D , E , F of a regular hexagon of side length 60 cm. Find the distance between the centre of gravity of this system and the centre of the hexagon.

9 A body of weight 3 newton is placed on a horizontal rough plane , the coefficient of friction between the plane and the body = $\frac{1}{3}$, and a horizontal force acts on the body trying to move it , then the force of friction $\in$ ....

(a) $[\frac{1}{3} , 3]$

(b) $]1 , \infty[$

(c) $]0 , 1]$

(d) $[0 , \frac{1}{3}]$

10 ABC is an equilateral triangle of side length 12 cm. A force of magnitude 7 newton acts along $\overrightarrow{AC}$, then the magnitude of its moment with respect to the point B equals .... newton.cm.

(a) 84

(b) $42\sqrt{3}$

(c) $12\sqrt{3}$

(d) 42

11 ***Answer one of the following items :***

[a] ABC is a triangle in which : AB = 3 cm. , BC = 5 cm. , m (∠ ABC) = 120° forces of magnitude 7.5 , 12.5 , 17.5 newton act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ respectively. Show that this system of forces is equivalent to a couple and find the norm of its moment.

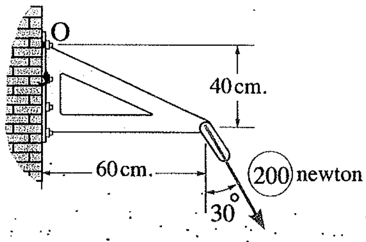
[b] The force $\vec{F}$ acts at the point A (– 3 , 2) If the moment of $\vec{F}$ about each of the points B (3 , 1) , C (– 1 , 4) equals 28 $\hat{k}$ Find $\vec{F}$

12 A uniform beam of length 80 cm. and weight 3 kg.wt. It is suspended at its two ends horizontally by two vertical strings each of them can not bear tension more than 5 kg.wt. Show where a mass of weight 4 kg.wt. can be suspend without any of the two strings has been cut.

13 A body of weight (w) is placed on a horizontal rough plane , the coefficient of friction between the plane and body equals tan α. the body is pulled by a string in a direction inclines with horizontal by an angle of measure θ. If the body becomes about to move. Prove that the tension in the string equals $w \sin \alpha \sec (\theta - \alpha)$.

14 **In the opposite figure :**

Find the moment of the force 200 newton with respect to the point O



15 If the two forces $\overrightarrow{F_1} = 3\hat{i} - a\hat{j}$, $\overrightarrow{F_2} = b\hat{i} + 2\hat{j}$ form a couple , then a – b = ...

(a) 5

(b) – 5

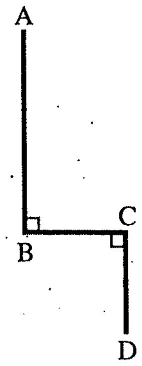
(c) – 1

(d) 1

16 **In the opposite figure :**

ABCD is a wire of length 32 cm. in which AB = 2 BC = 2 CD = 16 cm.

, then the distance between the centre of gravity of the wire from each of $\overleftrightarrow{BC}$, $\overleftrightarrow{BA}$ respectively is ....



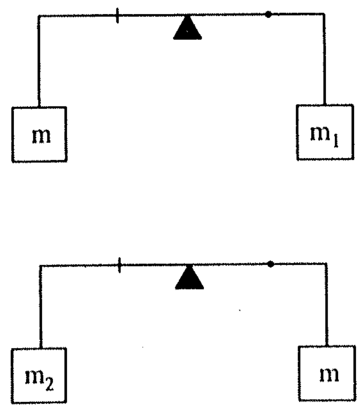
(a) (3 , 3)

(b) (4 , 4)

(c) (3 , 5)

(d) (4 , 8)

17 A light rod of length ℓ rests horizontally on a support as shown in the figure. If the mass m will be in equilibrium with each of the two masses m_1 and m_2 as shown in the figure , then the value of m in terms of m_1 and m_2 equals ....



(a) $m_1 + m_2$

(b) $\frac{1}{2}(m_1 + m_2)$

(c) $m_1 m_2$

(d) $\sqrt{m_1 m_2}$

18 $\overline{AB}$ is a rod of length 50 cm. and of weight 10 kg.wt acting at its midpoint. The rod can rotate easily in a vertical plane about a hinge fixed at A , a couple of moment 125 kg.wt.cm acts on the rod in a vertical plane. Prove that the reaction of the hinge at A equals the weight of the rod and also find the measure of the angle of inclination of the rod with the horizontal in the equilibrium position.

Model 18

Answer the following questions :

1 The distance between the centre of gravity of a thin uniform lamina in the shape of an equilateral triangle of side length 12 cm. from one of the vertices of the triangle equals .... cm.

(a) $2\sqrt{2}$
(b) $4\sqrt{3}$
(c) 6
(d) $6\sqrt{3}$

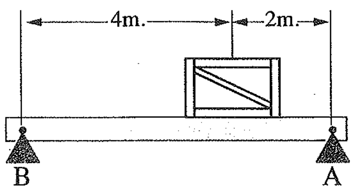
2 The following figures show two bricks equal in mass and volume, are placed on a horizontal rough plane in two different situations. A force F act on them to make them about to move, then ....

(a) $F_1 < F_2$
(b) $F_1 > F_2$
(c) $F_1 = F_2$
(d) no comparison between them.

3 If $\overrightarrow{F} = 5\hat{i} + \ell\hat{j} - \hat{k}$ acts at the point A (1 , – 2 , 3) and the moment of $\overrightarrow{F}$ with respect to the point B (– 2 , 2 , 4) equals $-5\hat{i} - 2\hat{j} - 7\hat{k}$, then ℓ = ...

(a) – 9
(b) – 2
(c) 3
(d) 9

4 The opposite figure shows a uniform wooden board of mass 30 kg. for each metre of its length. It rests horizontally on two supports A and B , it carries a box of mass 240 kg. , then the pressure happened on the support A equals .... kg.wt.



(a) 270
(b) 170
(c) 250
(d) 420

ABCD is a trapezium in which AB = 12 cm. , CD = 6 cm. , DA = 8 cm. , m (∠ A) = m (∠ D) = 90° , forces of magnitudes 12 , 18 , 15 , 9 newton in the directions $\overrightarrow{DA}$, $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ respectively prove that the system of forces is equivalent to a couple and determine the norm of its moment.

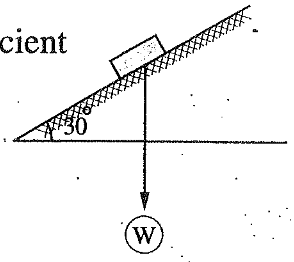
Answer one of the following items :

[a] $\hat{i}$ and $\hat{j}$ are two perpendicular unit vectors in the directions of the co-ordinate axes $\overrightarrow{Ox}$, $\overrightarrow{Oy}$. The two forces $\overrightarrow{F_1} = 2\hat{i} + a\hat{j}$, $\overrightarrow{F_2} = -6\hat{i} + 3\hat{j}$ are parallel. Determine the value of a , if the two forces act at the two points (1 , 0) , (5 , 0) respectively. Find the point of intersection of the line of action of their resultant with $\overrightarrow{Ox}$

[b] The force $\overrightarrow{F}$ acts in the plane of Δ ABC where A (3 , 2) , B (1 , – 4) , C (– 1 , 0) such that $\overrightarrow{M_A} = \overrightarrow{M_B} = 60\hat{k}$, $\overrightarrow{MC} = -60\hat{k}$ Find $\overrightarrow{F}$ and determine its magnitude.

7 **In the opposite figure :**

If the body is about to slide under its weight only , then the coefficient of friction between the body and the inclined plane = ...



(a) 30

(b) $\frac{\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}}{3}$

(d) $\frac{1}{2}$

8 ***Answer one of the following items :***

[a] Two forces of magnitudes 12 and F newton are parallel and their resultant is of magnitude 3 newton and its line of action is at a distance 30 cm. from the first force. Show that F has two values and find the distance between the two lines of action of the two forces.

[b] A body of weight 25 kg.wt. is placed on a rough plane inclined to the horizontal with an angle of sine = $\frac{3}{5}$ If the static coefficient of friction between the body and the plane equals $\frac{1}{5}$ Find the least force acting in a direction parallel to the plane to prevent the body from sliding.

9 A system consists of three masses distributed as follows : $m_1 = 1$ kg. at the point O (0 , 0) , $m_2 = 1$ kg. at the point N (3 , 0) , $m_3 = 2$ kg. at the point P (3 , 4) , then the centre of gravity of this system = ...

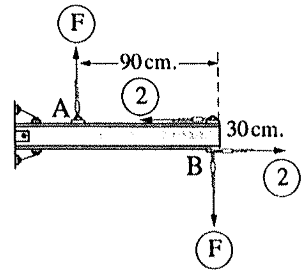
ⓐ $\left(\frac{9}{4}, 2\right)$

ⓑ $\left(2, \frac{9}{4}\right)$

ⓒ (0 , 2)

ⓓ $\left(\frac{9}{4}, 0\right)$

10 **In the opposite figure :**



If the moment of the resultant couple = – 1.5 newton.m.

, then F = ... newton.

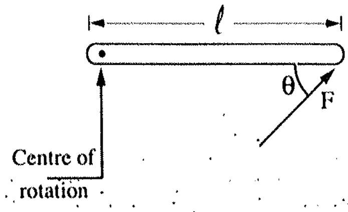
ⓐ 1

ⓑ $\frac{3}{7}$

ⓒ $\frac{13}{20}$

ⓓ $\frac{7}{3}$

11 **In the opposite figure :**



A rod of length ℓ can rotate easily about a point at one of its ends

A force F acts on the other end and inclines with the rod with an angle of measure θ If $\vec{F}$ should be perpendicular to the rod , then at any distance from the centre of rotation , F can act such that it has the same moment ?

ⓐ $\ell \sin \theta$

ⓑ $\ell \cos \theta$

ⓒ ℓ

ⓓ $\ell \tan \theta$

12 A force of magnitude $50\sqrt{2}$ gm.wt. acts along $\overrightarrow{AB}$ where A = (2 , 5) , B = (4 , 3) , then the magnitude of the moment of this force about the origin point = ...

ⓐ 200

ⓑ 350

ⓒ 100

ⓓ 150

(13) A body of weight 6 newton is placed on a rough horizontal plane. Two forces of magnitudes 2 , 4 newton and include between them an angle of measure 120° act on the body, then the body stays in equilibrium. Prove that the measure of the angle of friction λ between the body and the plane should not be less than 30 If $\lambda = 45°$ and the direction of the two forces does not change and the force 4 newton does not change. Determine the magnitude of the other force so that the body becomes about to move also determine the direction that the body is about to move in it.

14 ABCD is a rectangle in which AB = 8 cm. , BC = 6 cm. Forces of magnitudes 12 , 10 , F , k newton act along $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$ respectively. If the algebraic sum of the moment of these forces about each of the points C and M vanishes where M is the centre of the rectangle. Find the value of each of F and K

15 $\overline{AB}$ is a uniform ladder of length 5 metre and of weight 20 kg.wt. It rests with its end A on a vertical smooth wall and rests at the end B on a horizontal rough ground, the coefficient of friction between them = $\frac{1}{4}$, B is at distance 3 metre from the wall. Prove that the ladder will not be in equilibrium in this case. Then find the least weight of a body which the coefficient of friction between it and the ground = $\frac{1}{5}$ such that if it is placed at B it will prevent the ladder from sliding.

16. If $\overrightarrow{F_1} // \overrightarrow{F_2}$ and their resultant is $\overrightarrow{R}$ where $\overrightarrow{F_2} = -9\hat{i} + 12\hat{j}$, $\overrightarrow{R} = -2\overrightarrow{F_1}$, then $\overrightarrow{F_1} =$ ....

(a) $-15\hat{i} + 20\hat{j}$

(b) $-3\hat{i} + 4\hat{j}$

(c) $3\hat{i} - 4\hat{j}$

(d) $15\overrightarrow{i} - 20\overrightarrow{j}$

17. ABCD is a uniform lamina in thickness and density and it is in the shape of a rectangle in which AB = 12 cm. , BC = 16 cm. , E is the point of intersection of its diagonals $\overline{AC}$, $\overline{BD}$, the Δ AED is separated and fixed above Δ BEC. Find the centre of the lamina in this case with respect to E. If the lamina is suspended freely at C

Find the tangent of the angle of inclination of $\overrightarrow{CB}$ with the vertical line.

18 ABCD is a rectangular lamina where AB = 60 cm. , BC = 80 cm. and its weight is 40 newton. acting at the point of intersection of its diagonals. The lamina is suspended by a thin horizontal nail near by the vertex D such that its plane is vertical. If a couple of magnitude of its moment is 1000 newton.cm. and its direction is perpendicular to the plane of the lamina acted on it. Find the magnitude of reaction of the nail at D also find the measure of the angle of inclination of $\overline{DB}$ to the vertical line in the equilibrium position.

Model 19

Answer the following questions :

1 A body of weight 6 newton is placed on an inclined rough plane. It becomes about to slide. If the force of limiting friction = $3\sqrt{3}$ newton , then the measure of the inclination angle of the plane at the horizon = ...

(a) 30°

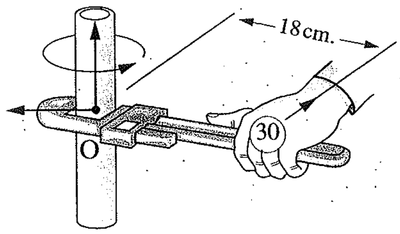
(b) 60°

(c) 45°

(d) zero°

2 **In the opposite figure :**

The magnitude of the moment of the force about the point O equals .... moment unit.



(a) 0.6

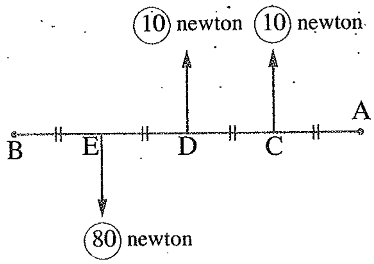
(b) 48

(c) 540

(d) 12

3 **In the opposite figure :**

The point of action of the resultant of the forces belongs to ....



(a) $\overline{AC}$

(b) $\overline{CD}$

(c) $\overline{DE}$

(d) $\overline{EB}$

4 The coefficient of friction is ....

(a) a force acting in the opposite direction of the force which acts on the body.

(b) the resultant of the normal reaction and the friction force.

(c) the ratio between the magnitude of the limiting friction and the magnitude of the normal reaction.

(d) the ratio between the magnitude of the resultant reaction and the limiting friction.

5. ABCDEF is a regular hexagon of side length 8 cm. Forces of magnitudes 8 , 1 , 8 , 6 , 5 gm.wt. along $\overrightarrow{AB}$, $\overrightarrow{DC}$, $\overrightarrow{DE}$, $\overrightarrow{BE}$, $\overrightarrow{FA}$ respectively. Prove that the system is equivalent to a couple and find the magnitude of its moment.

6. A body of weight 32 kg.wt. is about to slide under the action of its weight if it is placed on an inclined rough plane , where tangent of the inclination angle with the horizon = $\frac{1}{4}$ If this body is placed on a horizontal plane of the same roughness of the inclined plane and a tension force acts on the body upwards where it makes with the horizon an angle whose sine = $\frac{4}{5}$ to make the body about to move , find the magnitude of this force and the magnitude of the normal reaction.

7 ABC is a right-angled triangle at B where : AB = 9 cm. , BC = 12 cm. A force $\overrightarrow{F}$ acts in the plane of the triangle such that the algebraic measure of the moment of $\overrightarrow{F}$ about A equals the algebraic measure of the moment of $\overrightarrow{F}$ about C = 36 newton.cm. the algebraic measure of the moment of the force $\overrightarrow{F}$ about B = – 72 newton.cm. Determine the magnitude and the direction of the line of action of $\overrightarrow{F}$

8 If F_1 = 30 newton , F_2 = 70 newton and the distance between $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ equals 100 cm. where $\overrightarrow{F_1} \parallel \overrightarrow{F_2}$ and act in opposite directions , then the distance between the point of acting of the resultant and $\overrightarrow{F_1}$ equals .... cm.

(a) 175

(b) 30

(c) 70

(d) 40

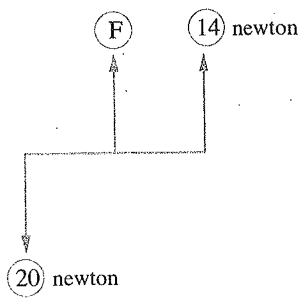
9 If the system of forces shown in the opposite figure is equivalent to a couple , then F = ... newton.

(a) 6

(b) 14

(c) 20

(d) 34



10 A body formed from a solid cylinder with radius length = r and its height = r , above it there is a hemisphere of radius length = r , then the centre of gravity of the body is ....

(a) inside the cylinder.

(b) inside the hemisphere.

(c) on the surface between the cylinder and the hemisphere.

(d) outside each of them.

11 ***Answer one of the following items :***

[a] Two parallel forces act in the same direction , their magnitudes are 3 F , F acting at the two points A and B respectively. If the force 3 F displaced such that it stays parallel to itself a distance X length unit on the ray $\overrightarrow{BA}$. Prove that the resultant of the two forces displace a distance = $\frac{3}{4}X$ in the same direction.

[b] The forces $\overrightarrow{F_1} = 3\hat{i} - \hat{j}$, $\overrightarrow{F_2} = -9\hat{i} + 3\hat{j}$ act at the points A (– 1 , 0) , B (1 , 2) respectively. Find the resultant of the two forces and determine the point of its action.

12 ***Answer one of the following items :***

[a] A uniform lamina in thickness and density in the shape of a circular disk its centre is the origin and its radius length = 6 length units. Two circular discs are cut and separated from it , the centre of one of them (– 1 , – 3) and its radius length = one length unit and the centre of the other is (1 , 2) and its radius length = 3 length unit. Find the centre of gravity of the remained part of the disc.

[b] $\overline{AB}$ is a uniform rod of length 90 cm. and its mass is 5 kg. , C and D are two points of its two trisection points from end A , masses of magnitudes 1 , 2 , 3 and 4 kg. are placed at A , B , C and D respectively. Determine the distance from the centre of gravity of the system to end A

13. $\overline{AB}$ is a uniform rod of length 120 cm. and weight 4 kg.wt. acting at its midpoint. It can rotate easily in vertical plane about a horizontal nail fixed and passes through a hole in the rod at a distance 20 cm. from B. If the rod rests by its end A on a smooth horizontal plane. Find the reaction of each of the horizontal plane and the nail on the rod. If the end B is pulled by a force perpendicular to the rod in the same vertical plane till the reaction of the horizontal plane equal to the weight , find this force and the magnitude and the direction of the reaction of the nail in this case given that the rod inclines to the horizontal with an angle of measure 30°

14 $\overline{AB}$ is a non-uniform rod of weight 4 kg.wt. and of length 100 cm. rests horizontally on two supports C and D such that AC = BD = 20 cm. If the pressure on the support C is twice the pressure on the support D. Determine the distance between the point of action of the weight of the rod from A , then find the weight which must be suspended at B such that the rod becomes about to rotate.

15 If $\overrightarrow{M_1}$ and $\overrightarrow{M_2}$ are two couples in equilibrium and $\overrightarrow{M_1} = 20\,\hat{k}$, then $\overrightarrow{M_1} - \overrightarrow{M_2} =$ ....

(a) $\vec{0}$

(b) $20\,\hat{k}$

(c) $40\,\hat{k}$

(d) $-40\,\hat{k}$

16 The force $\vec{F} = 3\,\hat{i} + \hat{j} - 2\,\hat{k}$ acts at the point A = (1 , − 1 , 2), then the length of the perpendicular drawn from the point B = (3 , − 2 , 1) on the line of action of the force $\vec{F}$ equals .... length unit.

(a) $\sqrt{10}$

(b) $\frac{1}{2}\sqrt{10}$

(c) $\sqrt{5}$

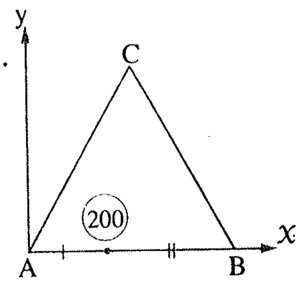
(d) $2\sqrt{5}$

17 $\overline{AB}$ is a uniform ladder of length 520 cm. and of weight 24 kg.wt. rests at its end A on a smooth horizontal plane and at its end B rests on a smooth vertical wall the ladder is kept in equilibrium by a string is connected at one of its ends to the base of the wall vertically down B and the other end is fixed at one of stairs of the ladder at a distance 130 cm. from A. If the end B is at a distance 480 cm. from the horizontal plane.

Find the magnitude of the reaction of each of the ground at A and the wall at B also find the magnitude of tension in the string.

18 **In the opposite figure :**

A uniform fine lamina , in thickness and density of mass 600 gm. in the shape of an equilateral triangle ABC of side length 36 cm. A mass of 200 gm. is fixed on the lamina at the point of trisection of $\overline{AB}$, then the centre of gravity of the system with respect to the two axes $\overrightarrow{AX}$, $\overrightarrow{AY}$ is ....



(a) $(16.5 , 4.5\sqrt{3})$

(b) $(6 , 4.5\sqrt{2})$

(c) $(18 , 4.5\sqrt{2})$

(d) $(16.5 , 13.5\sqrt{3})$

Model 20

Answer the following questions :

1. If the coefficient of friction between a body and a plane = 2 cos 60° , then the measure of the angle of friction = ...

   (a) 30°
   (b) 45°
   (c) 60°
   (d) 120°

2. If the forces $\overrightarrow{F_1} = -3\hat{i} + \hat{j}$, $\overrightarrow{F_2} = -\hat{i} + \hat{j}$, $\overrightarrow{F_3} = \hat{i} + 2\hat{j}$ act at the point A = (1 , 2) , then the length of the perpendicular drawn from the origin point to the line of action of the resultant of these forces = ... length unit.

   (a) 2
   (b) 10
   (c) 5
   (d) 15

3. $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$ are two parallel forces acting in opposite directions at the two points A and B respectively , $F_1 > F_2$, if their resultant $\overrightarrow{R}$ is of magnitude 10 newton and acts at C where $C \in \overleftrightarrow{AB}$ where AB = 8 cm. , AC = 12 cm. , then F_2 = ... newton.

   (a) 25
   (b) 20
   (c) 15
   (d) 10

4. The two equal forces which act on steering wheel of the car and causing rotation to the steering wheel are ....

   (a) friction.
   (b) couple.
   (c) a force perpendicular to the steering wheel.
   (d) non-zero resultant.

5 *Answer one of the following items :*

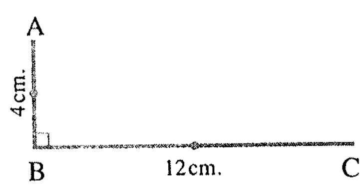
[a] A regular fair lamina in thickness and density in the shape of a square ABCD of side length 48 cm. M is the point of intersection of its diagonals. Δ CMD is cut and separated to be fixed on Δ CMB such that $\overline{MD}$ coincides $\overline{MB}$.

Find the distance between the centre of gravity of the lamina in this case from $\overline{BA}$ and $\overline{BC}$

[b] The masses 6.5 , 3.5 , 1.5 and 8.5 gm. are fixed at the vertices A , B , C and D of the rhombus ABCD in which AC = 2 BD = 16 cm.

Prove that the centre of gravity of these masses is at a distance $\sqrt{5}$ cm. from the centre of the rhombus.

6 The opposite figure represents a uniform wire in density and thickness where AB = 4 cm. , BC = 12 cm. , ∠ B is a right angle. If the wire is suspended freely from B , then the tangent of the angle between $\overleftrightarrow{BC}$ and the vertical in the equilibrium situation equals ....



(a) $\frac{1}{9}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) 3

7. The force $\overrightarrow{F_1} = 2\hat{i} - 3\hat{j}$ acts at the point A (2 , 3) and the force $\overrightarrow{F_2} = 4\hat{i} - 6\hat{j}$ acts at the point B (– 1 , – 3) , then the resultant acts at the point ....

(a) (1 , 0)

(b) (0 , – 1)

(c) (3 , – 1)

(d) (– 1 , 0)

8. ABCD is a rectangle in which AB = 30 cm. , BC = 40 cm. the forces of magnitudes 12 , 24 , 12 and 24 kg.wt. acts in the directions $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$, $\overrightarrow{DA}$ respectively. Prove that these forces are equivalent to a couple and find the magnitude of its moment , then find the magnitude of each of the two forces which act at A and C parallel to $\overleftrightarrow{BD}$ and make the system in equilibrium.

9. $\overline{AB}$ is a rod of length 180 cm. and weight 70 newton acting at its midpoint. the rod rests horizontally on a support at its end B and it is kept in equilibrium by a light vertical string fixed at a point of the rod at a distance 60 cm. from A The rod carries a weight 20 newton at a point at a distance 15 cm. from A. Determine the magnitude of the tension in the string.

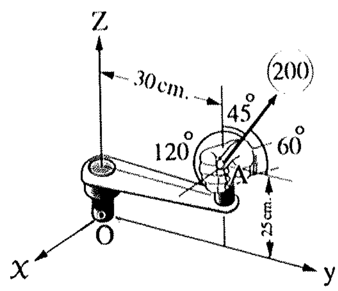
10. If the force $\vec{F}$ acts in the plane of the parallelogram ABCD and $M_A = -18$ moment unit , $M_B = M_D = 32$ moment unit. , then $M_C =$ .... moment unit.

(a) 50
(b) 82
(c) 46
(d) 14

11. In the opposite figure :

The moment of the force F = 200 newton about z-axis = ... newton. metre

(a) -25
(b) 30
(c) $30\sqrt{2} - 25$
(d) 60



12) A number (n) of coplanar and parallel forces are equal in magnitude each of them = F
They act in a direction parallel to y-axis the forces are consequently opposite in direction.
The first one act in the positive direction of y-axis at a distance 2 cm. from it.
The distance between each consecutive forces is 2 cm. If n is an odd number. Prove that
the algebraic sum of the moments of these forces about the origin point = $(n + 1) \times F$

13) $\overline{AB}$ is a uniform rod of weight 6 kg.wt. and of length 120 cm. It is connected with a hinge fixed on a vertical wall at the end A , a weight 8 kg.wt. is suspended at a point of the rod at a distance 30 cm. from the end A , the rod is kept in equilibrium horizontally by a light robe fixed at B and connected with a point on the wall lies vertically up A justly. If the robe inclines with the horizon by an angle of measure 30° , find the magnitude of the tension in the robe and the magnitude of the total reaction of the hinge.

14 If the norm of the moment of a couple is 350 newton.metre. and the norm of one of its forces = 70 newton , then the arm length of the couple = ...

(a) 50 metre.

(b) 5 metre.

(c) 5 cm.

(d) 24500 cm.

15 ***Answer one of the following items :***

[a] A body of weight (w) is placed on a plane inclining with the horizon with an angle of measure 30° , the coefficient of friction between the body and the plane equals $\frac{1}{3}$, find the magnitude of the horizontal force directed towards the plane and its line of action lies in the vertical plane which passes by the line of the greatest slope of the plane to act on the body to make it about to move downwards the plane.

[b] A body of weight (w) is placed on a horizontal rough plane. The angle of friction between it and the plane is of measure λ , the body is pulled with a string making an angle of measure θ upwards the horizontal.

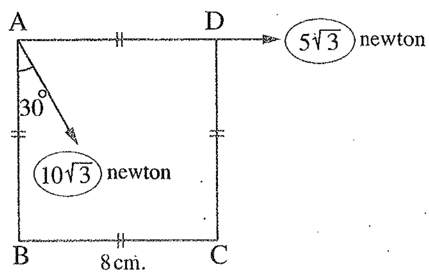
Prove that the force required to make the body about to begin motion equals $\frac{w \sin \lambda}{\cos (\lambda - \theta)}$

16 ABC is a lamina in the shape of an equilateral triangle of side length $8\sqrt{3}$ cm. and its weight = 300 gm.wt. act at the point of intersection of the medians of the triangle. If the lamina is suspended by a horizontal nail in a hole near by the vertex A , such that its plane is vertical and a couple of moment = 1200 gm.wt.cm. in magnitude acts in the plane of the lamina.

Find in the equilibrium position the measure of the angle of inclination of $\overline{AB}$ with the horizontal.

17 **In the opposite figure :**

The sum of moments of the shown forces about the point C equals .... newton.cm.



(a) $40\sqrt{3}$

(b) $120 - 80\sqrt{3}$

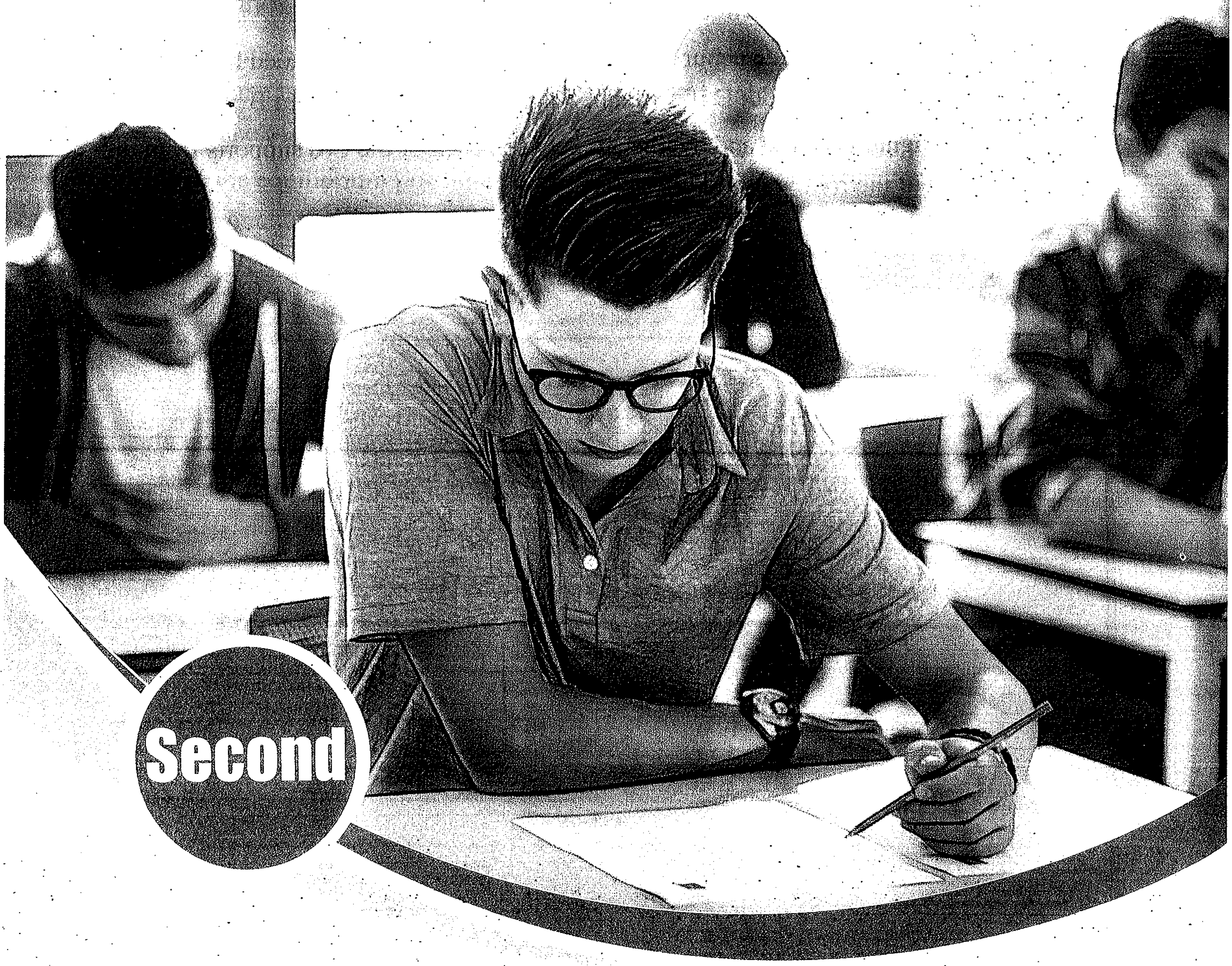
(c) $80\sqrt{3}$

(d) $120\sqrt{3}$

(18) $\overline{AB}$ is a uniform rod of length 80 cm. Rests in a horizontal position on two supports C and D where the distant between them 30 cm. , C is near to A

If the rod is about to rotate about C , if a weight 15 kg.wt. is suspended from A and is about to rotatat about D if a weight 5 kg.wt. is suspended from B

Find the weight of the rod and the distant between each of the two supports from the nearest end of the rod.



School Book Examinations

School Book Examinations

Model 1

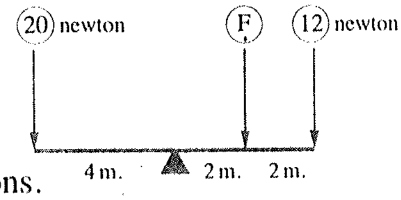
First : Answer the following question :

1 Choose the correct answer :

(1) If θ is the measure of the angle between the limiting friction force and the resultant reaction , then the coefficient of the static friction is equal to ....

(a) $\tan \theta$ (b) $\sin \theta$ (c) $\cos \theta$ (d) $\cot \theta$

(2) The opposite figure represents a rod in equilibrium , then F = ...



(a) 28 newtons. (b) 16 newtons.

(c) 2 newtons. (d) 4 newtons.

(3) The force $\vec{F} = 3\hat{i} - 5\hat{j}$ acts at point A (− 1 , 1) , then the moment of the force $\vec{F}$ about the origin point is equal to ....

(a) $-2\hat{k}$ (b) $2\hat{k}$ (c) $8\hat{k}$ (d) $-8\hat{k}$

(4) Two forces form a couple , the magnitude of one of them is 15 newtons and the moment of the couple is 45 newton.cm. , then the perpendicular distance between them is equal to ....

(a) 675 cm. (b) 60 cm. (c) 3 cm. (d) 30 cm.

(5) If a system of a coplanar forces is in equilibrium , then the algebraic measure of the sum of its moments about any point in the plane is equal to ....

(a) non-zero constant. (b) zero.

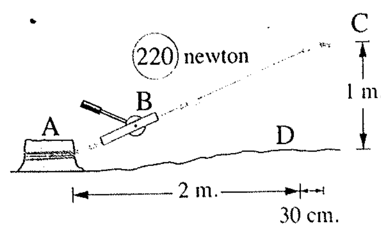
(c) resultant of these force. (d) the unity.

(6) The centre of gravity of two physical bodies of masses 3 kg. and 6 kg. and the distance between them is 15 cm. is at distance .... cm. from the 3 kg. body.

(a) 5 (b) 10 (c) 7.5 (d) 9

Second : Answer three questions of the following :

2 [a] The opposite figure illustrates a winch puller $\overline{AB}$ acting on an inclined fence $\overline{CD}$ Find the magnitude of the moment of the tension force about point D



«175.44 newton.metre»

[b] A body of weight (W) is placed on a rough plane inclined at an angle of measure (θ) to the horizon. If the measure of the angle of friction is (λ) , find the magnitude and direction of the least force making the body about to move upwards. «W sin (θ + λ)»

---

3 [a] Two like forces each of magnitude 10 , 15 newtons act at the two points A , B where AB = 75 cm. Find the resultant of the two forces. «25 newtons , 45 cm. far from A»

[b] ABC is an isosceles triangle in which AB = AC = 13 cm. and BC = 10 cm. forces of magnitudes 65 , F , 65 newtons act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ respectively. If the system is equivalent to a couple , what is the value of F and the magnitude of the moment of the couple ? «50 newtons , 600 newtons.cm.»

---

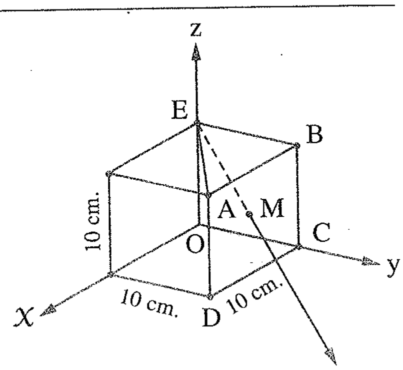
4 [a] AB is a light fine rod of length 2 ℓ connected in a vertical plane at its two ends A , B by two strings inclined at 30° , 60° to the vertical respectively , two weights of magnitudes 2 and 8 newtons. are suspended on the rod at distant $\frac{1}{5}\ell$, $\frac{6}{5}\ell$. from A. Find in the position of equilibrium the tension magnitude in the two strings and the measure of the angle of inclination of the rod to the horizontal. «$5\sqrt{3}$, 5 newtons , 30°»

[b] ABC is an equilateral triangle of side length 10 cm. the weights 3 , 6 , 9 act at its vertices A , B , C respectively. Identify the position of the centre of gravity of the system with respect to the point C «$\left(4\frac{1}{6} , \frac{5\sqrt{3}}{6}\right)$»

---

5 [a] **In the opposite figure :**

A force of magnitude $25\sqrt{6}$ newton acts at $\overrightarrow{EM}$ where M is the geometric centre of the square ABCD Find the components of the moment of the force about the coordinate axes.



«– 500 , 250 , zero»

[b] A fine lamina of uniform density in the form of a rectangle ABCD in which : AB = 5 cm. , BC = 12 cm. , E ∈ $\overline{AD}$ such that AE = 5 cm. The triangle ABE is bent about side $\overline{BE}$ until $\overline{AB}$ lies on $\overline{BC}$ totally. Identify the centre of gravity of the lamina after bending it with respect to $\overrightarrow{CB}$, $\overrightarrow{CD}$ «$\frac{407}{72}$, $\frac{155}{72}$»

Model 2

First : Answer the following question :

1 Choose the correct answer :

(1) Two couples act on a body , the magnitude of one of the two forces of the first couple is 20 kg.wt. the moment arm is $\frac{1}{2}$ m. and the direction of its rotation is in anti clockwise direction while the magnitude of one of the two forces of the second couple is 30 kg.wt. moment arm is 1 m. and the direction of its rotation is in the clockwise direction then the resultant couple is equal to ....

(a) 20 kg.wt.m. and the direction of its rotation is in the clockwise direction.

(b) 20 kg.wt.m. and the direction of its rotation is in the anti-clockwise direction.

(c) 40 kg.wt.m. and the direction of its rotation is in the clockwise direction.

(d) 40 kg.wt.m. and the direction of its rotation is in the anti-clockwise direction.

(2) The angle of friction is ....

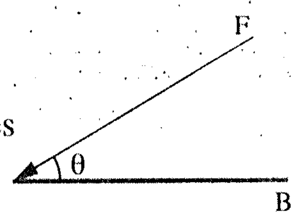
(a) the angle included between the normal reaction and the resultant reaction in the case of limiting friction.

(b) the ratio between the force of limiting friction and the normal reaction.

(c) the ratio between the coefficients of static and kinetic friction.

(d) the angle included between the force of the limiting friction and the resultant reaction.

(3) The opposite figure illustrates a force of magnitude F acts on an end of a rod , then the measure of angle θ which generates the maximum moment about points B is ....



(a) 0°　　(b) 90°　　(c) 45°　　(d) 30°

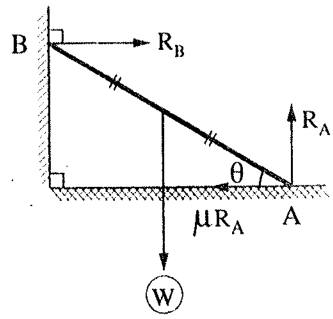
(4) Two unlike forces , the magnitude of one of them is 7 newtons and the magnitude of their resultant is 10 newtons , then the magnitude of the other force is ....

(a) 3 newtons.　　(b) 17 newtons.　　(c) 27 newtons.　　(d) 6 newtons.

(5) **In the opposite figure :**

If λ is the angle of friction between the ground and the rod , then

tan θ . tan λ = ...



(a) 2　　(b) $\frac{1}{2}$

(c) 1　　(d) 3

(6) A mass of 5 kg. acts at the point (2 , – 1) and a mass of 7 kg. acts at the point (1 , 2) , then the centre of gravity of the two masses acts at the point ....

(a) (17 , 9)　　(b) $\left(\frac{17}{12} , \frac{3}{4}\right)$　　(c) (19 , 13)　　(d) $\left(\frac{19}{12} , \frac{1}{4}\right)$

Second : Answer three questions of the following :

2 [a] If the force $\overrightarrow{F} = 5\hat{i} - \hat{j} + 3\hat{k}$ acts at the point A (– 1 , 2 , 1)

Find : (1) The moment of the force $\overrightarrow{F}$ about the origin point.

(2) The length of the perpendicular drawn from the origin point on the line of action of $\overrightarrow{F}$ «$7\hat{i} + 8\hat{j} - 9\hat{k}$, 2.35 length unit»

[b] Prove that if a body is placed on a rough inclined plane , and the body is about to slide , then the measure of the angle of friction is equal to the measure of the angle of inclination of the plane on the horizontal.

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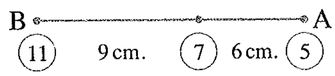
3 [a] Three bodies of weights 5 , 7 , 11 kg.wt. are placed on a light rod as shown in the figure. Identify the suspension point on the rod such that the rod remains horizontal.

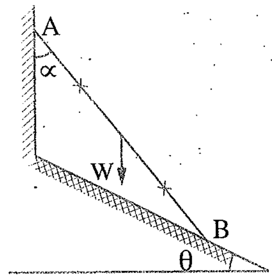


«9 cm. far from the point A»

[b] ABCD is a rectangle in which AB = 6 cm. , BC = 8 cm. , E $\in \overline{BC}$, G $\in \overline{AD}$ such that : BE = DG = 6 cm. The forces of magnitudes 5 , 5 , 7 , 7 , F , F kg.wt. act at the directions of $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{BC}$, $\overrightarrow{DA}$, $\overrightarrow{EA}$, $\overrightarrow{GC}$, respectively if the system is equivalent to a couple whose moment magnitude is 10 kg.wt.cm. in the direction of CBAD **Find :** F «$46\sqrt{2}$ kg.wt.»

---

4 [a] **In the opposite figure :**



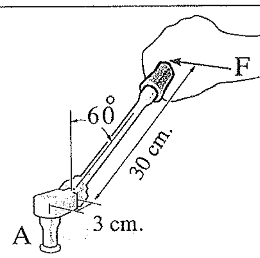
The top of a uniform ladder weight (W) is leaning against a smooth vertical wall and its base is leaning against a rough ground inclined to the horizontal at an angle of measure (θ) upwards. If the ladder is about to slide while it is in a vertical plane perpendicular to the intersection line of the wall and ground , prove that the ladder is inclined at an angle of measure ∝ to the vertical where tan ∝ = 2 tan (λ – θ) where λ is the angle of friction.

[b] A uniform rod $\overline{AC}$ of the length 15 ℓ is bent from point B where $\overline{AB} = 5\ \ell$ such that m (∠ ABC) = 90° and a rod is suspended freely from end A

Prove that $\overline{BC}$ is inclined at an angle of θ where $\tan \theta = \frac{4}{5}$ to the horizontal.

---

5 [a] **In the opposite figure :**



If the moment of the force F perpendicular to the rotation arm about point A is equal to 620 newtons.cm.

Find : F «19 newtons»

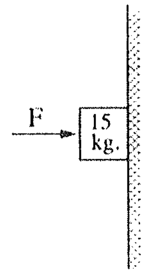
[b] ABC is an equilateral triangle of side length 20 cm. , M is the intersection point of its medians , D is the midpoint of $\overline{BC}$, masses of magnitudes 15 , 30 , 75 , 45 , 45 are fixed at the points A , B , D , C , M respectively. Identify the centre of gravity of this system. Where does the centre of gravity of the remaining system lie if the mass existed at B is lifted? «$\left(\frac{65}{7}, \frac{10\sqrt{3}}{7}\right)$, $\left(\frac{15}{2}, \frac{5\sqrt{3}}{3}\right)$ from the point C»

Model 3

First : Answer the following question :

1 Complete :

(1) The magnitude of the least horizontal force $\overrightarrow{F}$ needed to equilibrate a body of mass 15 kg.wt. on a rough vertical plane the coefficient of the static friction between it and the body is equal to $\frac{1}{5}$ is equal to .... kg.wt.

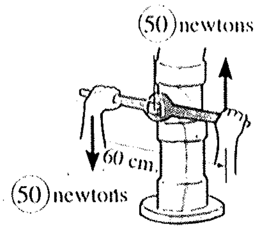


(2) A force of magnitude 70 newtons acts at $\overrightarrow{AB}$ where ABCD is a square of side length 10 cm. , then the magnitude of the moment of the force about the centre of the square is equal to ....

(3) If $\overrightarrow{F_1} \,//\, \overrightarrow{F_2}$, $\overrightarrow{F_1} = \hat{i} - 2\hat{j}$, $\|\overrightarrow{F_2}\| = 4\sqrt{5}$ unit , then $\overrightarrow{F_2} =$ ....

(4) **In the opposite figure :**

The moment of the couple resulted from the two forces 50 , 50 is equal to ....



(5) When a rod is placed in a smooth spherical container , it is in equilibrium when the line of action of weight passes ....

(6) The centre of gravity of the triangular uniform lamina lies at ....

Second : Answer three questions of the following :

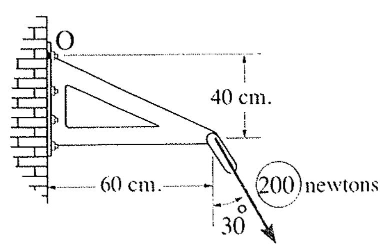
2 [a] A body of weight $66\frac{2}{3}$ newtons is placed on a rough horizontal plane. The coefficient of friction between them is equal to $\frac{3}{4}$

A force of magnitude 40 newtons acts on the body and inclined at an acute angle of measure θ to the horizontal plane. What is the value of θ if the body is about to move ?

«36° 52`»

[b] **In the opposite figure :**

Find the moment of the force 200 newtons about O



«– 6392.3 newtons. cm.»

3 [a] $\overline{AB}$ is a non uniform rod of length 1 m. it rests in a horizontal position on two supports at C , D where AC = 20 cm. , BD = 10 cm. If the heaviest weight can be suspended at point A or point B without disturbing the rod is 5 , 4 kg.wt. respectively. Find the weight of the rod and its point of action. «2 kg.wt. , 70 cm. far from the point A»

[b] ABCDEF is a uniform hexagon of side length 10 cm. The forces of magnitudes 2 , 5 , 4 , 6 , 1 , 3 newtons acts at $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{ED}$, $\overrightarrow{EF}$, $\overrightarrow{AF}$ respectively.
Find the magnitude and direction of the force which should act at the centre of the hexagon in order to reduce the system to a couple , then identify its moment.
«$\sqrt{3}$ newton in direction $\overrightarrow{CA}$, $-35\sqrt{3}$ newton.cm.»

---

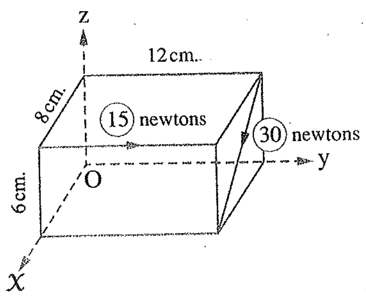
4 [a] $\overline{AB}$ is a uniform rod of weight (W) is leaning by one of its ends A on smooth horizontal ground and its end B on a smooth plane inclined to the horizontal at an angle twice the measure of the angle of inclination of the rod to the horizontal in an equilibrium. The rod in being kept by a string one of its ends is connected to the end of the rod leaning on the horizontal ground and the other end of the string is in a point on the intersection line of the inclined plane with the horizontal plane. Find the magnitude of tension in the string and the two reactions at the two ends of the rod when the rod inclines at 30° to the horizontal. «$\frac{\sqrt{3}}{4}$ W , $\frac{3}{4}$ W , $\frac{W}{2}$»

[b] ABCD is a fine uniform lamina in the form of a square of side length ℓ If E , F , N are the midpoints of $\overline{AB}$, $\overline{AD}$, $\overline{BC}$ respectively. The triangle AEF is bent about $\overline{EF}$ such that A is coincident on the centre of the square M and the triangle BEN on $\overline{EN}$ such that the vertex B is coincident on the centre of the square M
Identify the centre of gravity of the lamina in its new position.
« with respect to the centre of the square $\left(-\frac{1}{24}\ell , 0\right)$ »

---

5 [a] **In the opposite figure :**

Find the sum of moments of the forces about O



«$-306\,\hat{i} + 144\,\hat{j} - 168\,\hat{k}$»

[b] **Find the centre of gravity for the following distribution :**

W_1 = 20 newtons acts at (2 , 1) , W_2 = 15 newtons acts at (– 3 , 1) , W_3 = 25 newtons act at (1 , – 1) «$\left(\frac{1}{3} , \frac{1}{6}\right)$»

Model 4

First : Answer the following question :

1 Choose the correct answer :

(1) The coefficient of friction is based upon ....

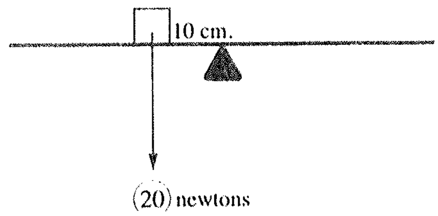
(a) the area of the contact surface between two bodies.

(b) shape of the two bodies.

(c) nature of the two bodies.

(d) all mentioned.

(2) The opposite figure represents a uniform rod leaning on a support at its midpoint. A body is placed on the rod as shown in the figure which of the following forces makes the rod be in equilibrium ....



(a) a force of magnitude 10 newtons upwards act on a distance 20 cm. on the right of the rod midpoint.

(b) a force of magnitude 10 newtons downwards acts on a distance 20 cm. on the right of the rod midpoint.

(c) a force of magnitude 30 newtons upwards acts on a distance 5 cm. on the left of the rod midpoints.

(d) a force of magnitude 30 newtons downwards act on a distance 5 cm. on the left of the rod midpoint.

(3) The force $\overrightarrow{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$ acts at point A whose position vector with respect to the origin point is $A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$, then the component of moment of $\overrightarrow{F}$ about x-axis is ....

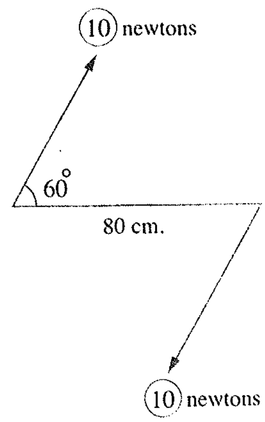
(a) $F_x \times A_z - F_z \times A_x$

(b) $-F_y \times A_z + F_z \times A_y$

(c) $F_x \times A_y + F_y \times A_x$

(d) $F_y \times A_y + F_z \times A_z$

(4) The magnitude of the moment of the opposite couple is equal to ....



(a) 800 newtons cm.

(b) 400 newtons cm.

(c) $400\sqrt{3}$ newtons cm.

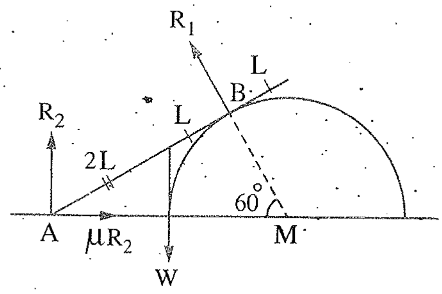
(d) $-400\sqrt{3}$ newtons cm.

(5) In the opposite figure :

If the rod is about to slide

, then R_1 = ...

, R_2 = ...



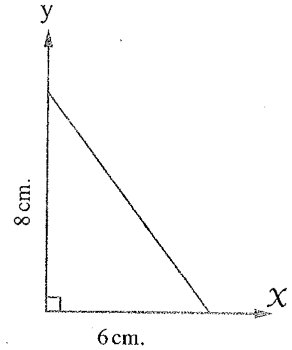
(a) W

(b) $\frac{1}{2}$ W

(c) $\frac{\sqrt{3}}{2}$ W

(d) $\frac{\sqrt{3}}{3}$ W

(6) The centre of gravity of the regular density shaded lamina in the opposite figure is ....



(a) $\left(2 , \frac{8}{3}\right)$

(b) $\left(\frac{8}{3} , 2\right)$

(c) (6 , 8)

(d) (8 , 6)

Second : Answer three questions of the following :

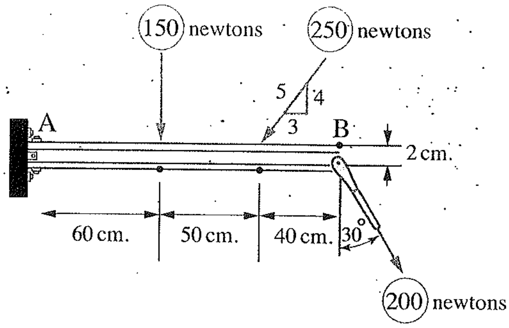
2 [a] A body of weight 16 kg.wt. is placed on a plane inclined at 30° to the horizontal and the coefficient of friction between it and the body is equal to $\frac{1}{\sqrt{3}}$

A force in the line of the greatest slope of the plane acts on the body upwards and with magnitude 10 kg.wt. If the body is in equilibrium , identify the friction force and show whether the body is about to move or not. «2 kg.wt. , is not about to move»

[b] In the opposite figure :

Three coplanar forces act on the rod $\overline{AB}$

Find the algebraic measures of the sum of the moments of the forces about each of the two points A , B



«21700 kg.wt.cm.»

3 [a] A uniform rod of length 4 metres rests at its midpoint. Two weights of 4 , 3 kg.wt. are suspended in one of its two halves distant 1 , 1.5 m. from its midpoint respectively and two other weights of 5 kg.wt. , W kg.wt. are suspended in the other half distant $\frac{1}{2}$, 2 m. from its midpoint respectively. What is the value of W if the rod is in equilibrium ? «3 kg.wt.»

[b] ABC is a uniform lamina in the form of an equilateral triangle of side length $30\sqrt{3}$ cm. and weight 50 gm.wt. the lamina is suspended by a horizontal pin from a hole close to vertex A to be vertically in equilibrium. A couple perpendicular to the surface of the lamina acts on the lamina to be in equilibrium in a position $\overline{AB}$ is horizontal.

Find the moment of the couple acting and the reaction of the pin.

«$750\sqrt{3}$ gm.wt.cm. , 50 gm.wt.»

4 [a] $\overline{AB}$ is a uniform rod. Its end A is connected to a hinge fixed in a vertical wall and end B is connected by an end of a string and the other end of the string is connected by a point in the horizontal plane passing through the hinge such that each of the rod and the string inclined at an angle of measure θ to horizontal. If (W) is the weight of the rod , show that the reaction of the hinge at A is equal to $\frac{W}{4}\sqrt{8 + \csc^2 \theta}$

[b] ABCD is a square of side length 20 cm. Four masses equal in magnitudes are placed at its vertices.

(1) Identify the centre of gravity of the system.

(2) Where does the centre of gravity of the remaining system lie if the mass placed at one of the vertices is ceased ? «$(10 , 10)$, $\left(\frac{20}{3} , \frac{20}{3}\right)$»

---

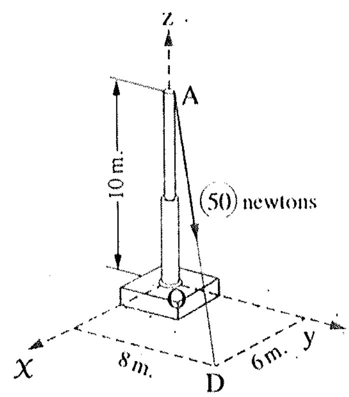
5 [a] ABC is a regular lamina in the form of an equilateral triangle whose weight is 3 kg. and M is its centre of gravity. Forces of magnitudes 2 , 2 , 11 kg. are placed at the vertices A , B , C respectively.

Prove that the centre of gravity of the system lies at the midpoint of $\overline{MC}$

[b] In the opposite figure :

A force of magnitude 50 newtons acts at point A

Find the moment of the force about point O



«$-200\sqrt{2}\,\hat{i} + 150\sqrt{2}\,\hat{j}$»

Model 5

First : Answer the following question :

1 Complete :

(1) The coefficient of the static friction is the ratio between ....

(2) If the force $\vec{F} = 2\hat{i} - \hat{j} + 5\hat{k}$ acts at point A whose position vector is $\hat{i} - 3\hat{k}$, then the moment of $\vec{F}$ about point B whose position vector is $\hat{j} + 3\hat{k}$ is equal to ....

(3) Two like forces , the magnitude of one of them is twice the magnitude of the second and the magnitude of their resultant is 31 newtons , then the magnitude of the smaller force is equal to ....

(4) If the two forces $\vec{F_1} = A\hat{i} + 5\hat{j}$, $\vec{F_2} = 3\hat{i} - B\hat{j}$ form a couple , then $A + B =$ ....

(5) The necessary and sufficient condition for the equilibrium of a system of coplanar forces ....

(6) The centre of gravity of the rigid body suspended freely on the vertical straight line passing through ....

Second : Answer three questions of the following :

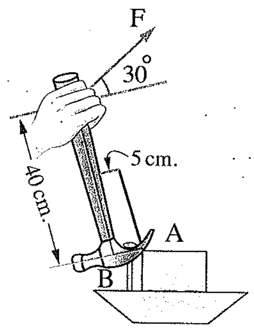
2 [a] A body of weight 50 newtons is placed on a rough inclined plane inclined at an angle of measure θ to the horizontal. The least and greatest forces parallel to the line of the greatest slope and makes the body in equilibrium on the plane are 10 , 40 newtons respectively. Find the coefficient of friction and the measure of the angle of inclination of the plane to the horizontal.

«$\frac{\sqrt{3}}{5}$, 30°»

[b] The opposite figure illustrates the force F needed to remove a nail at B

If the magnitude of the moment of the force about point A needed to remove the nail is equal to 200 newton cm.

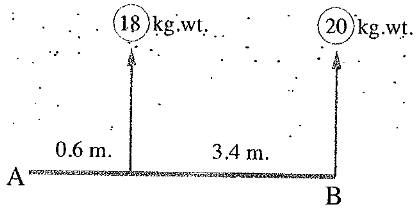
Find the magnitude of the force F



«5.38 newton»

---

3 [a] If the resultant of three forces act on the rod $\overline{AB}$ of negligible weight in the figure is 13.6 kg.wt. and acting upwards distant 3 metres on the right of A Find the magnitude direction and point of action of the third force.



«24.4 kg.wt. and 2.05 m. downward from the point A»

[b] ABCD is a rectangle which AB = 12 cm. , BC = 9 cm. , M $\in \overline{BC}$ such that BM = 4 cm. forces of magnitudes F_1 , $8\sqrt{10}$, 26 , F_2 , 18 newton in the direction $\overrightarrow{BA}$, $\overrightarrow{AM}$, $\overrightarrow{DC}$, $\overrightarrow{DA}$ respectively. If the system of forces is in equilibrium , find the value for each of F_1 , F_2

«24 , 24 newton»

---

4 [a] $\overline{AB}$ is a uniform ladder of length 5 m. and weight 20 kg.wt. rests with its end A on a smooth vertical wall and on rough horizontal ground with its end B and the coefficient of fricition between them is $\frac{1}{4}$ and the end B is at a distant 3 m. from the wall. Prove that the ladder is not in equilibrium in this case , then find the smallest weight of the body which the coefficient of friction between it and the ground is $\frac{1}{5}$ such that if it is placed at the end B of the ladder , it stops the ladder from sliding.

«$12\frac{1}{2}$ kg.wt.»

[b] A uniform wire of length 100 cm. is bent in the form of five sides of a uniform hexagon ABCDEF and starts from point A , Identify the distance between its centre of gravity and that of the hexagon. If the wire is freely suspended from end A , identify the measure of the angle of inclination of $\overline{AB}$ to the vertical in the equilibrium state.

« $2\sqrt{3}$, 55° 42´»

---

5 [a] $\overline{AB}$ is a uniform rod of length 2 m. and weight 5 newtons C , D are two trisection points from direction A , weights of magnitudes 1 , 2 , 3 , 4 newtons are connected at point A , C , D , B respectively. Identify the centre of gravity of the system.

« $\frac{11}{9}$ m. from the point A»

[b] Two forces $\overrightarrow{F_1} = 2\hat{i} - \hat{j}$, $\overrightarrow{F_2} = \hat{j} - 2\hat{i}$ act at two points A (1 , 1) , B (0 , – 4) respectively. Find the moments of the system about any point on the plane. « $11\hat{k}$»

Guide Answers of

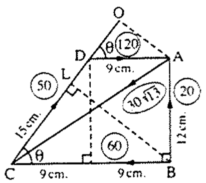
Egypt Exams and Model Examinations

Answers of Egypt exams

Answers of first session 2017

1 ⓑ

2



From the figure :

$BL = 18 \sin \theta = 18 \times \frac{12}{15}$

$= 14.4$ cm.

$AO = 9 \sin \theta = 9 \times \frac{12}{15}$

$= 7.2$ cm.

, $AC = 6\sqrt{13}$ cm.

$M_A = -60 \times 12 - 50 \times 7.2 = -1080$ gm.wt.cm. (1)

, $M_B = 30\sqrt{13} \times \frac{12 \times 18}{6\sqrt{13}} - 120 \times 12 - 50 \times 14.4$

$= -1080$ gm.wt.cm. (2)

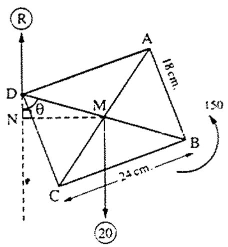
, $M_C = 20 \times 18 - 120 \times 12 = -1080$ gm.wt.cm. (3)

From (1) , (2) , (3) :

$\therefore M_A = M_B = M_C = -1080$ gm.wt.cm.

∴ The system is equivalent to a couple whose moment $= -1080$ gm.wt.cm.

3



$\therefore BD = \sqrt{(24)^2 + (18)^2}$

$= 30$ cm.

∴ MD = 15 cm.

∴ MN = 15 sin θ

∵ The lamina is in equilibrium under action of two couples

∴ The two forces (R , 20) form a couple

, the algebraic measure of its moment

$= -150$ newton.cm.

$\therefore -150 = -20 \times MN$ $\therefore -150 = -20 \times 15 \sin \theta$

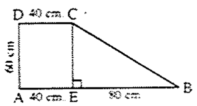
$\therefore \sin \theta = \frac{1}{2}$ $\therefore \theta = 30°$ or $150°$

∴ The measure of angle of inclination of $\overline{BD}$ to vertical downward equals 30° or 150°

4 ⓑ

5 ⓐ

6



The area of rectangle AECD : the area of triangle EBC = 40 × 60 : $\frac{1}{2} \times 80 \times 60 = 1 : 1$

Let the mass of the rectangle = m and acts at the point (20 , 30)

∴ The mass of the triangle = m

and acts at the point $\left(\frac{40 + 40 + 120}{3} , \frac{0 + 60 + 0}{3}\right)$

$= \left(\frac{200}{3} , 20\right)$

The mass	m	m
x	20	$\frac{200}{3}$
y	30	20

$\therefore x_G = \frac{20\,m + \frac{200}{3}\,m}{2\,m} = \frac{130}{3}$

, $y_G = \frac{30\,m + 20\,m}{2\,m} = 25$

∴ The distance from the centre of gravity of the lamina and each of $\overline{AD}$ and $\overline{AB}$ are $\frac{130}{3}$ cm. and 25 cm.

7 ⓐ

8 ⓒ

9

[a] $\overrightarrow{M_B} = \overrightarrow{BA} \times \overrightarrow{F} = (\overrightarrow{A} - \overrightarrow{B}) \times \overrightarrow{F}$

$= (-1 , 2 , 3) \times (2 , 3 , -1)$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ 2 & 3 & -1 \end{vmatrix} = -11\hat{i} + 5\hat{j} - 7\hat{k}$

∴ The length of perpendicular

$= \frac{\|\overrightarrow{M_B}\|}{\|\overrightarrow{F}\|} = \frac{\sqrt{(-11)^2 + (5)^2 + (-7)^2}}{\sqrt{(2)^2 + (3)^2 + (-1)^2}}$

$= 3.73$ length unit.

[b] ∵ $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = 4\hat{i} - 3\hat{j}$

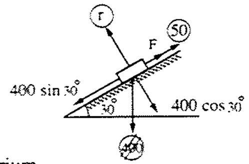
, $\overrightarrow{M_B} = \overrightarrow{BA} \times \overrightarrow{R} = (\overrightarrow{A} - \overrightarrow{B}) \times \overrightarrow{R}$

$= (-4 , -2) \times (4 , -3) = 20\hat{k}$

, $l = \frac{\|\overrightarrow{M_B}\|}{\|\overrightarrow{R}\|} = \frac{20}{\sqrt{16 + 9}} = 4$ length unit.

10 ⓑ **11** ⓐ

12



∵ 400 sin 30° > 50

∴ $\overrightarrow{F}$ acts in the direction of the line of the greatest slope upwards

, ∵ the body is in equilibrium

∴ F + 50 = 400 sin 30°

∴ F = 150 gm.wt.

, ∵ $r = 400 \cos 30° = 200\sqrt{3}$

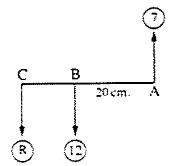
$\therefore \mu_s r = \frac{\sqrt{3}}{4} \times 200\sqrt{3} = 150$ gm.wt.

$\therefore F = \mu_s r$ ∴ The body is about to move

13 ⓐ

14 ⓓ

15



R = 12 − 7 = 5 newton.

, 7 × AC = 12 (AC − 20)

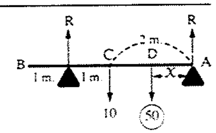
∴ 7 AC = 12 AC − 240

∴ 5 AC = 240 ∴ AC = 48 cm.

∴ R = 5 newton. and acts in the direction of the force 12 newton. and its action point is at a distance 48 cm. from A

16 ⓒ

17



From the equilibrium

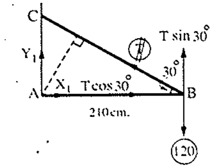
2 R = 60 ∴ R = 30

, ∵ M_A = Zero

∴ $50 \times x + 10 \times 2 = 30 \times 3$

∴ $x = 1.4$ m.

18



[a] From the figure :

, ∵ The rod is in equilibrium

$\therefore X_1 = T \cos 30°$

$\therefore X_1 = \frac{\sqrt{3}}{2} T$ (1)

, $Y_1 + T \sin 30° = 120$ $\therefore Y_1 + \frac{1}{2} T = 120$ (2)

, ∵ M_A = zero

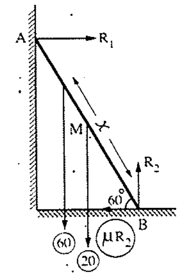
∴ T × 210 sin 30° = 120 × 210

∴ T = 240 newton.

From (1) : $\therefore X_1 = \frac{\sqrt{3}}{2} \times 240 = 120\sqrt{3}$ newton.

, from (2) : $Y_1 = 120 - \frac{1}{2} \times 240 =$ zero

$\therefore R = 120\sqrt{3}$ newton in the direction $\overrightarrow{AB}$



[b] Assuming that the girl ascends up the ladder a distance (x)

the equations of equilibrium

$R_1 = \frac{1}{2\sqrt{3}} R_2$, $R_2 = 80$

$R_1 = \frac{1}{2\sqrt{3}} \times 80 = \frac{40\sqrt{3}}{3}$

, M_B = zero

$\therefore 20 \times \frac{l}{2} \cos 60° + 60\,x \cos 60° - R_1 \times l \sin 60°$

$= 0$

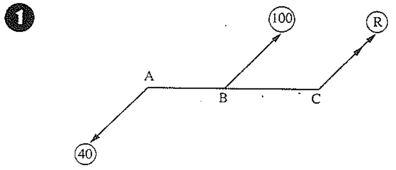
$\therefore 5\,l + 30\,x = \frac{40\sqrt{3}}{3}\,l \times \frac{\sqrt{3}}{2}$

$\therefore 30\,x = 20\,l - 5\,l = 15\,l$ $\therefore x = \frac{1}{2}\,l$

i.e. The maximum distance that the girl can ascend up the ladder is $\frac{1}{2}$ the length of the ladder.

Answers of second session 2017

1



$\therefore R = F_2 - F_1$ $\quad\quad \therefore R = 100 - 40 = 60$ newton

$\therefore 40 \times AC = 100\,(AC - 240)$

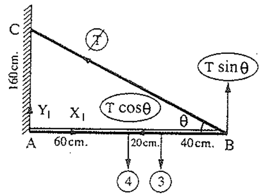
$40\,AC = 100\,AC - 24000$ $\quad\quad \therefore AC = 400$ cm.

$\therefore$ Point of action of the resultant distant 400 cm. from A and 160 cm. from B

2 ⓓ

3

[a]



$BC = \sqrt{(120)^2 + (160)^2} = 200$ cm.

The equilibrium equations : $\therefore X_1 - T\cos\theta = 0$

$\therefore X_1 = \frac{3}{5}T$ (1)

$, T\sin\theta + Y_1 - 3 - 4 = 0$

$\therefore \frac{4}{5}T + Y_1 = 7$ (2)

$M_A =$ zero

$\therefore T \times \frac{4}{5} \times 120 - 3 \times 80 - 4 \times 60 = 0$

$\therefore T = 5$ newton

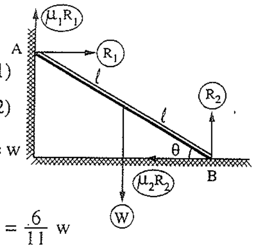
From (1) : $\therefore X_1 = 3$ newton

, from (2) : $\therefore Y_1 = 3$ newton

$\therefore R = \sqrt{9+9} = 3\sqrt{2}$ newton

$\tan\theta = 1$ $\quad\quad \therefore \theta = 45°$ with the horizontal.

[b] The equilibrium equations



$\therefore R_1 = \frac{3}{4}\mu_2$ (1)

$, R_2 + \frac{1}{2}R_1 = w$ (2)

$\therefore R_2 + \frac{1}{2}\left(\frac{3}{4}R_2\right) = w$

$\therefore \frac{11}{8}R_2 = w$

$\therefore R_2 = \frac{8}{11}w \;,\; R_1 = \frac{6}{11}w$

$\because M_B = 0$

$\therefore w\,\ell\cos\theta - R_1 \times 2\,\ell\sin\theta - \frac{1}{2}R_1 \times 2\,\ell\cos\theta = 0$

[dividing by $(\ell\cos\theta)$]

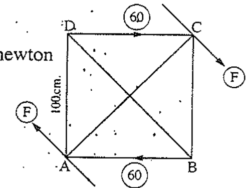
$\therefore w - 2R_1\tan\theta - R_1 = 0$ (3)

$, \because R_1 = \frac{6}{11}w$

$\therefore w - \frac{12\,w}{11}\tan\theta - \frac{6}{11}w = 0$

$\therefore \tan\theta = \frac{5}{12}$

4



The two forces (60 , 60) newton Form a couple

The algebraic measure of its moment

$M_1 = -60 \times 100 = -6000$ newton.cm.

$\because$ The two forces F , F form a couple.

The algebraic measure of its moment

$M_2 = F \times AC = -100\sqrt{2}\,F$

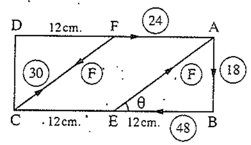
$\because M_1 = M_2$

$\therefore -100\sqrt{2}\,F = -6000$

$\therefore F = \frac{6000}{100\sqrt{2}} = 30\sqrt{2}$ newton.

$\therefore$ The two forces $30\sqrt{2}$, $30\sqrt{2}$ newton.

5



$CF = \sqrt{12^2 + 9^2} = 15$ cm.

$\because$ The four forces act along sides of the quadrilateral ABCF in the cyclic ordered.

$, \because \frac{18}{9} = \frac{48}{24} = \frac{30}{15} = \frac{24}{12} = 2$

$\therefore$ The system of the forces equivalent to a couple its moment norm

$= 2 \times \left[\frac{1}{2}(12 + 24) \times 9\right] \times 2 = 648$ gm.wt.cm.

the system be equilibrium with two additional forces (F , F) acting along $\overrightarrow{EA}$, $\overrightarrow{FC}$ respectively

$\therefore F \times 12 \times \sin\theta = 648$

$\therefore F = \frac{648}{12 \times \frac{9}{15}} = 90$ gm.wt.

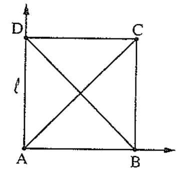
6 ⓒ

7 ⓓ

8

Let the length of the square = 1 cm.

	A	B	C	D
Mass	100	100	100	100
x	0	ℓ	ℓ	0
y	0	0	ℓ	ℓ



$x_G = \frac{100 \times 0 + 100\,\ell + 100\,\ell + 100 \times 0}{400} = \frac{1}{2}\,\ell$ cm.

$, y_G = \frac{100 \times 0 + 100 \times 0 + 100\,\ell + 100\,\ell}{400} = \frac{1}{2}\,\ell$ cm.

$\because$ The centre of gravity distance $\frac{1}{2}\,\ell$ from $\overleftrightarrow{AB}$, $\overleftrightarrow{AD}$

Another solution :

$\because$ The four masses are equal at the vertices of the square

$\therefore$ The centre of the gravity of the system lies on the geometrical centre of the square $\left(\frac{\ell}{2} , \frac{\ell}{2}\right)$

9 ⓒ

10 ⓑ

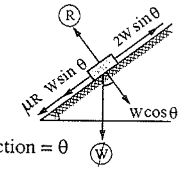
11

[a] The equilibrium equations $R = w\cos\theta$

$, 2\,w\sin\theta = w\sin\theta + \mu R$

$\therefore w\sin\theta = \mu\,(w\cos\theta)$

$\therefore \mu = \frac{\sin\theta}{\cos\theta} = \tan\theta$



$\therefore$ Measure of the angle of friction = θ

[b] The value of resultant reaction

$= \sqrt{R^2 + \mu^2 R^2}$

$= \sqrt{w^2\cos^2\theta + \tan^2\theta \times w^2\cos^2\theta}$

$= \sqrt{w^2(\cos^2\theta + \sin^2\theta)} = w$

12 ⓐ

13 ⓑ

14

[a] $\vec{r} = \overrightarrow{BA} = \vec{A} - \vec{B} = (-3 , 1 , 2) - (2 , 2 , -1)$

$= (-5 , -1 , 3)$

$\overrightarrow{M_B} = \overrightarrow{BA} \times \vec{F}$

$= (-5 , -1 , 3) \times (2 , -1 , 3)$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -1 & 3 \\ 2 & -1 & 3 \end{vmatrix}$

$=$ zero $\hat{i} + 21\,\hat{j} + 7\,\hat{k} = 21\,\hat{j} + 7\,\hat{k}$

length of perpendicular segment

$= \frac{\|\overrightarrow{M_B}\|}{\|\vec{F}\|} = \frac{\sqrt{21^2 + 7^2}}{\sqrt{2^2 + (-1)^2 + 3^2}} = \sqrt{35}$ length unit.

[b] $\because \overrightarrow{M_o} = -9\,\hat{k}$

$\therefore \overrightarrow{OA} \times \overrightarrow{F_1} + \overrightarrow{OB} \times \overrightarrow{F_2} + \overrightarrow{OC} \times \overrightarrow{F_3} = -9\,\hat{k}$

$(1 , 2) \times (\ell , m) + (0 , 4) \times (1 , -3) + (2 , 4)$

$\times (-2 , 1) = -9\,\hat{k}$

$\therefore (m - 2\,\ell)\,\hat{k} + (0 - 4)\,\hat{k} + (2 + 8)\,\hat{k} = -9\,\hat{k}$

$\therefore m - 2\,\ell = -15$ (1)

$\because \overrightarrow{M_D} = -4\,\hat{k}$

$\therefore \overrightarrow{DA} \times \overrightarrow{F_1} + \overrightarrow{DB} \times \overrightarrow{F_2} + \overrightarrow{DC} \times \overrightarrow{F_3} = -4\,\hat{k}$

$\therefore (3 , -1) \times (\ell , m) + (2 , 1) \times (1 , -3) + (4 , 1)$

$\times (-2 , 1) = -4\,\hat{k}$

$\therefore (3\,m + \ell)\,\hat{k} + (-6 - 1)\,\hat{k} + (4 + 2)\,\hat{k} = -4\,\hat{k}$

$3\,m + \ell = -3$ (2)

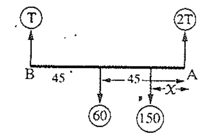
From (1) , (2) : we get $\ell = 6$, $m = -3$

15 ⓓ

16 ⓒ

17 ⓐ

18



$\because$ The system in equilibrium

$\therefore 2\,T + T = 60 + 150$

$\therefore 3\,T = 210$

$\therefore T = 70$ newton

$\overrightarrow{M_A} = 0$

$\therefore 150\,x + 60 \times 45 - 70 \times 90 = 0$

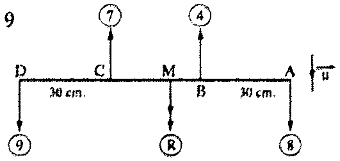
$\therefore x = 24$ cm.

$\therefore$ It is suspended at the point distant 24 cm. from A

Answers of first session 2018

1 ⓒ **2** ⓒ

3



$R = 8 - 4 - 7 + 9$

$= 6$ newton, acting in direction of 8 , 9 newton.

, let the point of action of the resultant is $M \in \overline{AD}$

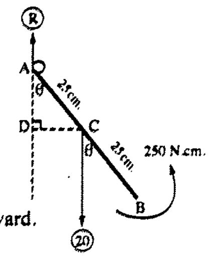
∵ Sum of moments of the forces about A = the moment of the resultant about A

$\therefore -4 \times 30 - 7 \times 60 + 9 \times 90 = 6 \times AM$

∴ AM = 45 cm. , M lies on the left of A

$\therefore M \in \overline{AD}$, MA = 45 cm.

4



∵ The rod is in equilibrium under acting two couples

∴ The two forces (20 , R) form a couple its algebraic measure is – 250 newton. cm.

∴ R = 20 newton, vertical upward.

$, -20 \times CD = -250$

∴ CD = 12.5

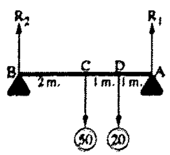
$\therefore \sin\theta = \frac{12.5}{25} = \frac{1}{2}$

$\therefore \theta = 30°$ or $150°$

∴ The rod inclined by angle 30° or 150° to the vertical.

5 ⓒ **6** ⓒ

7



The rod is in equilibrium

$\therefore R_1 + R_2 = 70$ (1)

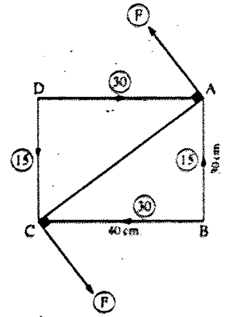
$, M_A = $ zero

$\therefore 20 \times 1 + 50 \times 2 - R_2 \times 4 = 0$

$\therefore R_2 = 30$ kg.wt.

From (1) : $R_1 = 40$ kg.wt.

8



∵ The two forces (30 , 30) form a couple its algebraic measure is M_1

$\therefore M_1 = -30 \times 30$

$= -900$ dyne.cm.

∵ The two forces (15 , 15) form a couple its algebraic measure is M_2

$\therefore M_2 = 15 \times 40 = 600$ dyne. cm.

$\therefore M = M_1 + M_2 = -300$ dyne. cm.

$\therefore \|\vec{M}\| = 300$ dyne. cm.

∵ From equilibrium state : $\vec{M} = -M$

$\therefore F \times 50 = 300$ ∴ F = 6 dyne.

∴ The two forces are 6 , 6 dyne.

9 ⓐ **10** ⓒ

11

[a] $\overrightarrow{BA} = \vec{A} - \vec{B} = (-1 , 2 , 3)$

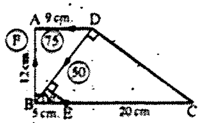
$$\therefore \overrightarrow{M_B} = \overrightarrow{BA} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ 2 & 3 & -1 \end{vmatrix}$$

$$= -11\hat{i} + 5\hat{j} - 7\hat{k}$$

The length of perpendicular from the point "B"

$$= \frac{\|\overrightarrow{M_B}\|}{\|\vec{F}\|} = \frac{\sqrt{(-11)^2 + (5)^2 + (-7)^2}}{\sqrt{(2)^2 + (3)^2 + (-1)^2}} = 3.73 \text{ length unit.}$$

[b] From the figure :



$DB = \sqrt{9^2 + 12^2} = 15$

$, DC = \sqrt{(25)^2 - (15)^2}$

$= 20$ cm.

$\because M_C = $ zero.

$\therefore 50 \times 20 + 75 \times 12 - F \times 25 = 0$

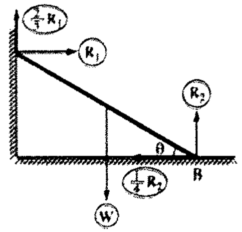
∴ F = 76 newton.

$, M_E = 50 \sin\theta \times 5 - 76 \times 5 + 75 \times 12$

$= 50 \times \frac{20}{25} \times 5 + 520 = 720$ newton. cm.

12 ⓑ **13** ⓐ

14



[a] From the equations of equilibrium :

$R_1 = \frac{1}{4} R_2$ (1)

$, R_2 + \frac{2}{3} R_1 = w$ (2)

From (1) , (2) we get :

$4 R_1 + \frac{2}{3} R_1 = w$

$\therefore \frac{14}{3} R_1 = w$ (3)

$, M_B = $ zero

$\therefore w \times \frac{l}{2} \cos\theta - R_1 \times l \sin\theta - \frac{2}{3} R_1 \times l \cos\theta = $ zero

(divided by $l \cos\theta$)

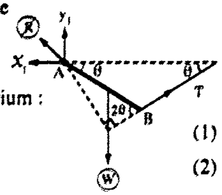
$\therefore \frac{w}{2} - R_1 \tan\theta - \frac{2}{3} R_1 = $ zero (4)

by substitution from (3) in (4) :

$\therefore \frac{7}{3} R_1 - R_1 \tan\theta - \frac{2}{3} R_1 = $ zero

$\therefore \tan\theta = \frac{5}{3}$ $\therefore \theta = 59° 2'$

[b] Let the components of the reaction of the hinge be X_1 and y_1



The equations of equilibrium :

$X_1 = T \cos\theta$ (1)

$y_1 = w - T \sin\theta$ (2)

$, M_A = $ zero

$\therefore T \times l \sin 2\theta = w \times \frac{l}{2} \cos\theta$

$\therefore T \times 2 \sin\theta \cos\theta = \frac{1}{2} w \cos\theta$

i.e. $T = \frac{1}{4} w \csc\theta$

substituting in (1) : $\therefore X_1 = \frac{1}{4} w \cot\theta$

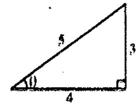
substituting in (2) : $\therefore y_1 = w - \frac{1}{4} w = \frac{3}{4} w$

∴ R (the reaction of the hinge) $= \sqrt{X_1^2 + y_1^2}$

$$\therefore R = \sqrt{\frac{1}{16} w^2 \cot^2\theta + \frac{9}{16} w^2} = \frac{1}{4} w \sqrt{\cot^2\theta + 9}$$

15 ⓐ **16** ⓒ

17

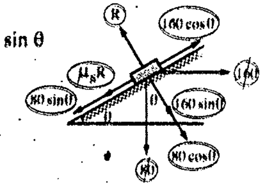


∵ The body is about to move upward :

$\therefore R = 80 \cos\theta + 160 \sin\theta$

$\therefore R = 80 \times \frac{4}{5} + 160 \times \frac{3}{5}$

$= 160$ newton



$, 160 \cos\theta = \mu_s R + 80 \sin\theta$

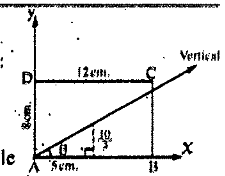
$\therefore 160 \times \frac{4}{5}$

$= \mu_s \times 160 + 80 \times \frac{3}{5}$

$\therefore 160 \mu_s = 80$

$\therefore \mu_s = \frac{1}{2}$

18



Area of the rectangle NLCE :

Area of the rectangle ABCD

$= 6 \times 4 : 12 \times 8 = 1 : 4$

∴ let the mass of the rectangle NLCE = m

, let the mass of the rectangle ABCD = 4 m

and let the two perpendicular directions $\overrightarrow{AB}$, $\overrightarrow{AD}$ are $\overrightarrow{ox}$, $\overrightarrow{oy}$

∴

Mass	– m	4 m
x	9	6
y	6	4

$$X_g = \frac{-9\,m + 4\,m \times 6}{-m + 4\,m} = 5$$

$$y_g = \frac{-6\,m + 4\,m \times 4}{-m + 4\,m} = \frac{10}{3}$$

∴ the centre of gravity of the remaining part $\left(5 , \frac{10}{3}\right)$

i.e. the centre of gravity distance from $\overrightarrow{AD}$ 5 cm. and from $\overrightarrow{AB}$ $\frac{10}{3}$ cm.

$$\therefore \tan\theta = \frac{\left(\frac{10}{3}\right)}{5} = \frac{2}{3}$$

Answers of second session 2018

1 ⓒ **2** ⓒ

3

$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3}$

$= 2\hat{i} - 4\hat{j} + \hat{i} - 3\hat{j} - 3\hat{i} + 7\hat{j} = \overrightarrow{O}$ (1)

, $\overrightarrow{M_O} = \overrightarrow{OA} \times \overrightarrow{F_1} + \overrightarrow{OB} \times \overrightarrow{F_2} + \overrightarrow{OC} \times \overrightarrow{F_3}$

$= (-1, 1) \times (2, -4) + (-2, 3) \times (1, -3)$

$+ (0, 1) \times (-3, 7)$

$= 2\hat{k} + 3\hat{k} + 3\hat{k} = 8\hat{k} \neq 0$ (2)

From (1) , (2) :

∴ The system is equivalent to a couple

, its algebraic measure = 8 moment unit.

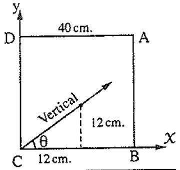
4

	A	B	C	E
Mass	5	10	15	20
x	40	40	0	0
y	40	0	0	20

$x_G = \frac{5 \times 40 + 10 \times 40 + 15 \times 0 + 20 \times 0}{5 + 10 + 15 + 20} = 12$

, $y_G = \frac{5 \times 40 + 10 \times 0 + 15 \times 0 + 20 \times 20}{5 + 10 + 15 + 20} = 12$

∴ The centre of gravity = (12 , 12)



$\tan\theta = \frac{12}{12} = 1$

∴ m (∠ θ) = 45°

i.e. $\overline{BC}$ inclines by an angle 45° with the vertical.

5 ⓐ **6** ⓓ

7

[a] $M_B = \overrightarrow{BA} \times \overrightarrow{F} = (-1, 1, -4) \times (3, -2, 4)$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -4 \\ 3 & -2 & 4 \end{vmatrix} = -4\hat{i} - 8\hat{j} - \hat{k}$

The length of the perpendicular from B

$= \frac{\| \overrightarrow{M_B} \|}{\| \overrightarrow{F} \|} = \frac{\sqrt{(-4)^2 + (-8)^2 + (-1)^2}}{\sqrt{(3)^2 + (-2)^2 + (4)^2}}$

$= \frac{9\sqrt{29}}{29} \simeq 1.67$ length unit.

[b] $M_C = 100 \times 20 - 80\sqrt{2} \sin 45° \times 25 = 0$

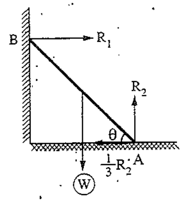
∴ The resultant passes through the point "C"

, $M_A = -80\sqrt{2} \sin 45° \times 45 = -3600$.

∴ The magnitude of the moments of the forces about "A" = 3600 N.cm.

8 ⓑ **9** ⓑ

10



[a] ∵ The rod is in equilibrium

∴ $R_2 = W$

, $R_1 = \frac{1}{3} R_2$

∴ $R_1 = \frac{1}{3} W$ (1)

, $\mu_A = 0$

∴ $W \times \frac{1}{2} L \cos\theta - R_1 \times L \sin\theta = 0$

From equation (1) :

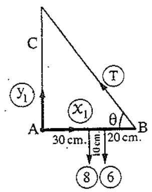
∴ $W \times \frac{1}{2} L \cos\theta = \frac{1}{3} W \times L \sin\theta$

∴ $\frac{1}{2} \cos\theta = \frac{1}{3} \sin\theta$

∴ $\frac{\sin\theta}{\cos\theta} = \frac{3}{2}$ ∴ $\tan\theta = \frac{3}{2}$

∴ m (∠ θ) ≃ 56° 19`

[b] ∵ The rod is in equilibrium :



∴ M_A = zero

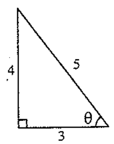
∴ $-8 \times 30 - 60 \times 40$

$+ T \sin\theta \times 60 = 0$

∴ $-480 + T \times \frac{4}{5} \times 60 = 0$

∴ T = 10 newton.

, $x_1 = T\cos\theta$



∴ $x_1 = 10 \times \frac{3}{5} = 6$ newton

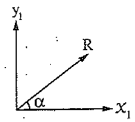
, $y_1 + T\sin\theta = 6 + 8$

∴ $y_1 + 10 \times \frac{4}{5} = 14$

∴ $y_1 = 6$ newton.

∴ $R = \sqrt{x_1^2 + y_1^2} = \sqrt{6^2 + 6^2}$

$= 6\sqrt{2}$ newton.

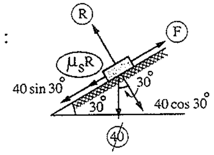


, $\tan\alpha = \frac{6}{6} = 1$

∴ m (∠ α) = 45°

11 ⓒ **12** ⓓ

13



∵ The body is about to move :

∴ R = 40 cos 30°

$= 20\sqrt{3}$ newton.

, $F = \mu_s R + 40 \sin 30°$

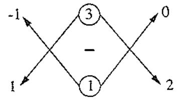
$= \frac{\sqrt{3}}{2} \times 20\sqrt{3} + 20 = 50$ newton.

14

$\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

$= 3\hat{i} - \hat{j} + (-9\hat{i}) + 3\hat{j}$

$= -6\hat{i} + 2\hat{j}$



, ∵ $\overrightarrow{F_2} = -3\overrightarrow{F_1}$

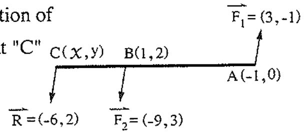
, let R acts at the point "C" which divides $\overline{AB}$ externally in the ratio 3 : 1

∴ $C = \left(\frac{3 \times 1 - 1 \times -1}{3 - 1}, \frac{3 \times 2 - 0}{3 - 1}\right)$

∴ C = (2 , 3)

Another solution :

Let the point of action of the resultant acts at "C"



∴ $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

$= -6\hat{i} + 2\hat{j}$

∴ $-6x = -9 \times 1 + 3 \times -1$

∴ $x = 2$

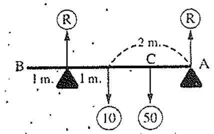
, $2y = 3 \times 2 - 1 \times 0$

∴ y = 3

∴ C = (2 , 3)

15 ⓑ **16** ⓒ

17



∵ The rod is in equilibrium

∴ 2 R = 50 + 10

∴ R = 30 kg.

, $M_A = 0$

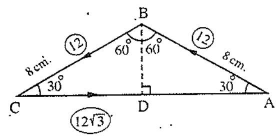
∴ $50 \times AC + 10 \times 2 - 30 \times 3 = 0$

∴ 50 AC = 70

∴ AC = 1.4 metre

i.e. The weight 50 kg.wt. should suspended at a distance 1.4 m. From A to equate the pressure on the 2 supports.

18



From the figure :

DB = 4 cm.

, $AD = 4\sqrt{3}$ cm.

, $AC = 8\sqrt{3}$ cm.

In Δ ABC :

$\frac{12}{8} = \frac{12}{8} = \frac{12\sqrt{3}}{8\sqrt{3}} = \frac{3}{2}$

, the forces act in the same way round.

∴ The system is equivalent to a couple its moment

$= 2 \times$ the area of $\Delta ABC \times \frac{3}{2}$

$= 2 \times \left[\frac{1}{2} \times 8\sqrt{3} \times 4\right] \times \frac{3}{2} = 48\sqrt{3}$ newton.cm.

Answers of model examinations

Model 1

1 ⓓ **2** ⓐ **3** ⓑ
4 ⓑ **5** ⓓ **6** ⓑ

7

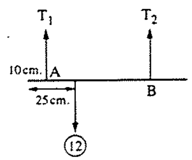
$M_A = 0$

$\therefore 12 \times 15 - 60\, T_2 = 0$

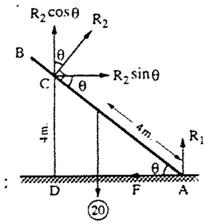
$T_2 = 3$ kg.wt.

$\therefore T_1 + T_2 = 12$

$T_1 = 12 - 3 = 9$ kg.wt.



8



$\frac{4}{AD} = \frac{4}{3}$

$\therefore AD = 3$ meter

$\therefore AC = 5$ meter

$\therefore CB = 3$ meter

$\therefore$ The equations of equilibrium :

$R_2 \sin\theta = F_s$

$\therefore \frac{4}{5} R_2 = F_s$ (1)

, $R_1 + R_2 \cos\theta = 20$ $\therefore R_1 + \frac{3}{5} R_2 = 20$ (2)

, $M_A = 0$ $\therefore 20 \times 4 \times \cos\theta - R_2 \times 5 = 0$

$80 \times \frac{3}{5} = 5\, R_2$

$\therefore 48 = 5\, R_2$ $\therefore R_2 = 9.6$ kg.wt.

, from (1) : $\therefore F_s = 7.68$

, from (2) : $\therefore R_1 = 14.24$ kg.wt.

$\therefore \mu_1 = \frac{F_s}{R_1} = \frac{7.68}{14.24} = \frac{48}{89}$

9

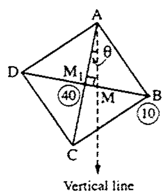
$10 \times MB = 40 \times M M_1$

$\therefore M M_1 = \frac{1}{4} MB$

$\therefore M M_1 = \frac{1}{5} M_1 B$

, $\because M_1 A = M B$

$\therefore M_1 M = \frac{1}{5} M_1 A$



$\therefore \frac{M_1 M}{M_1 A} = \frac{1}{5}$

In $\Delta A M_1 M$, let $m(\angle M A M_1) = \theta$

$\therefore \tan\theta = \frac{M_1 M}{M_1 A} = \frac{1}{5}$

$\therefore m(\angle\theta) = \tan^{-1}\left(\frac{1}{5}\right) \simeq 11° 19'$

10

[a] $\overrightarrow{M_O} = \overrightarrow{OA} \times \overrightarrow{F} = (4 , 2) \times (l , m)$

$= (4\,m - 2\,l)\,\hat{k} = -22\,\hat{k}$ (1)

$\overrightarrow{M_B} = \overrightarrow{BA} \times \overrightarrow{F} = (-9 , 1) \times (l , m)$

$= (-9\,m - l)\,\hat{k} = 22\,\hat{k}$ (2)

From (1) , (2) :

$m = -3$, $l = 5$

[b] $\overrightarrow{r} = \overrightarrow{BA} = (0 , 2)$

$\therefore \overrightarrow{M_B} = (0 \times -2 - 2 \times 1)\,\hat{k} = -2\,\hat{k}$

, $L = \frac{\|\overrightarrow{M_B}\|}{F} = \frac{2}{\sqrt{(1)^2 + (-2)^2}} = \frac{2\sqrt{5}}{5}$ length unit.

11 ⓑ

12

$\because R = 200 + F - K - 100 = 300$

$\therefore F - K = 200$ (1)

$\because$ The sum of the moment of the forces about A = the moment of the resultant about A

$\therefore K \times 20 - F \times 50 + 100 \times 70 = -300 \times 40$

$\therefore 2\,K - 5\,F = -1900$ (2)

From (1) , (2) :

F = 500 newton , K = 300 newton.

13

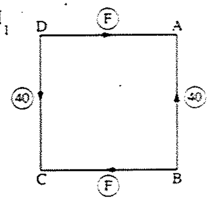
$\because$ The two forces (40 , 40) form a couple its algebraic measure of its moment is M_1

$\therefore M_1 = 40 \times 16$

$= 640$ gm.wt.cm.

$\because$ The two forces (F , F) form a couple its algebraic measure of its moment is (M_2)

$\therefore M_2 = -F \times 16 = -16\,F$ gm.wt.cm.



$M = M_1 + M_2$

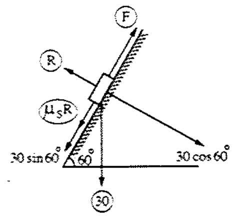
$\therefore 480 = 640 - 16\,F$

$\therefore 16\,F = 640 - 480 = 160$

$\therefore F = 10$ gm.wt.

14

[a]



$\because$ The body is about to slide down under the effect of its weight only when the plane inclined by angle 30° to the horizontal.

$\therefore \mu_s = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore$ The body is about to moves upwards

$\therefore F = \mu_s R + 30 \sin 60°$

, $R = 30 \cos 60° = 15$

$\therefore F = \frac{1}{\sqrt{3}} \times 15 + 30 \times \frac{\sqrt{3}}{2}$

$\therefore F = 20\sqrt{3}$ newton.

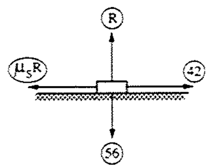
[b] $\because$ The body is about to move

$\therefore R = 56$

, $42 = \mu_s R$

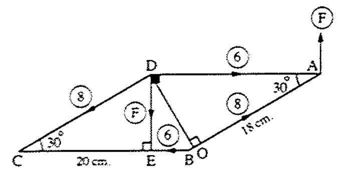
$\therefore 42 = \mu_s \times 56$

$\therefore \mu_s = \frac{3}{4}$



15 ⓒ **16** ⓒ **17** ⓐ

18



DE = 18 sin 30° = 9 cm.

, DO = 20 sin 30° = 10 cm.

$\because$ The two forces (6 , 6) form a couple , its algebraic measure is M_1 where :

$M_1 = -6 \times 9 = -54$ newton.cm.

$\because$ The two forces (8 , 8) form a couple , its algebraic measure is M_2 where :

$M_2 = 8 \times 10 = 80$ newton.cm.

$\therefore M = M_1 + M_2 = 26$ newton.cm.

$\|\overrightarrow{M}\| = 26$ newton.cm.

$\because$ The two forces (F , F) form a couple , its algebraic measure = 26 newton.cm.

$\therefore F \times 20 = 26$ $\therefore F = 1.3$ newton

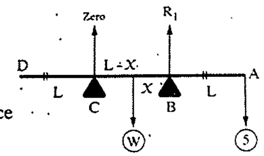
$\therefore$ The two forces are 1.3 and 1.3 newton

Model 2

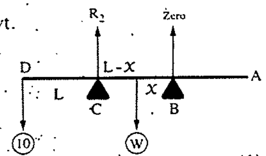
1 ⓒ **2** ⓑ **3** ⓒ **4** ⓑ **5** ⓐ

6

Let AB = BC = CD = L and the centre of gravity of the rod acts at a distance X from the point "B"



* When the body 5 kg.wt. is suspending from the point A



$\therefore R_2 = 0$

$\therefore M_B = 0 : 5 \times L = w\,X$ (1)

* When the body 10 kg.wt. is suspending from the end "D"

$\therefore R_1 = 0$

$\therefore M_C = 0 : 10 \times L = w\,(L - X)$ (2)

Divide (2) by (1) : $2 = \frac{L - X}{X}$

$\therefore 2\,X = L - X$ $\therefore X = \frac{1}{3} L$

From equation (1) : $5\,L = w \times \frac{1}{3} L$

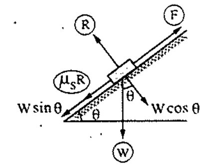
$\therefore w = 15$ kg.wt.

7 ⓑ **8** ⓓ

9

$\because$ The body is about to slide down under the effect of its weight only

$\therefore \mu_s = \tan\theta$



When the force "F" acts in the direction of the line of the greatest slope upwards.

$\therefore$ The body about to move upwards.

$\therefore F = \mu_s R + w \sin\theta$

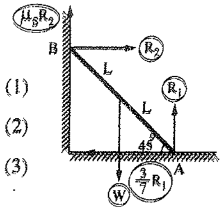
$, \because R = w\cos\theta$

$\therefore F = \tan\theta \times R + w \sin\theta$

$\therefore F = \frac{\sin\theta}{\cos\theta} \times w\cos\theta + w\sin\theta = 2w\sin\theta$

10 ⓑ **11** ⓑ

12



Let the length of the ladder = 2 L

$R_1 + \mu_s R_2 = w$ (1)

$R_2 = \frac{3}{7} R_1$ (2)

$\therefore R_1 + \frac{3}{7}\mu_s R_1 = w$ (3)

$M_A = 0$

$L \times w \times \cos 45° - 2L \times R_2 \sin 45°$

$- 2L \times \mu_s R_2 \cos 45° = 0$

$\therefore w - 2R_2 - 2\mu_s R_2 = 0$

From (2) :

$w - 2\left(\frac{3}{7}R_1\right) - 2\mu_s\left(\frac{3}{7}R_1\right) = 0$

$\therefore w = \frac{6}{7}R_1 + \frac{6}{7}\mu_s R_1$ (4)

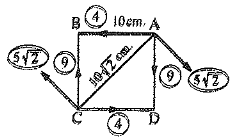
From (3) , (4) :

$\frac{6}{7}R_1 + \frac{6}{7}\mu_s R_1 = R_1 + \frac{3}{7}\mu_s R_1$

$\therefore \frac{3}{7}\mu_s = \frac{1}{7}$

$\therefore \mu_s = \frac{1}{3}$

13



The two forces (4 , 4) form a couple the algebraic measure of its moment is (M_1)

$M_1 = 4 \times 10 = 40$ newton.cm.

, the two forces (9 , 9) form a couple , its algebraic measure (M_2)

$M_2 = -9 \times 10 = -90$ newton.cm.

, the two forces $(5\sqrt{2} , 5\sqrt{2})$ form a couple , its algebraic measure (M_3)

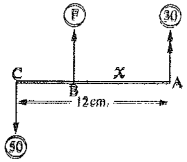
$M_3 = -5\sqrt{2} \times 10\sqrt{2} = -100$ newton.cm.

$M = M_1 + M_2 + M_3 = 40 + (-90) + (-100)$

$= -150$ newton.cm.

$\therefore$ The norm of moment = 150 newton.cm.

14



[a] $R = F - 50$

$\therefore 30 = F - 50$

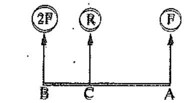
$\therefore F = 80$ newton.

$M_A = 50 \times 12 - 80 \times x = 0$

$x = 7.5$

$\therefore$ The distance between the two forces

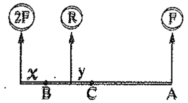
$= 12 - 7.5 = 4.5$ cm.



[b] Before moving :

$F \times AC = 2F \times BC$

$\therefore AC = 2BC$ (1)



After moving :

$F \times (AC + y) = 2F(BC - y + x)$

$\therefore AC + y = 2BC - 2y + 2x$ (2)

From (1) , (2) :

$2BC + y = 2BC - 2y + 2x$

$\therefore 3y = 2x$

$y = \frac{2}{3}x$

15

$A = (0 , 0 , 3)$, $B = (4 , 12 , 0)$, $D = (0 , 12 , 0)$

$\therefore \overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A} = (4 , 12 , -3)$

$, \overrightarrow{F} = F \times \frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} = 130 \times \frac{(4 , 12 , -3)}{\sqrt{(4)^2 + (12)^2 + (-3)^2}}$

$= (40 , 120 , -30)$

$, \because \overrightarrow{DA} = \overrightarrow{A} - \overrightarrow{D} = (0 , -12 , 3)$

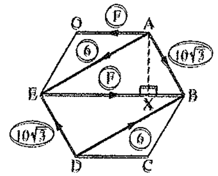
$\therefore \overrightarrow{M_D} = \overrightarrow{DA} \times \overrightarrow{F}$

$= (0 , -12 , 3) \times (40 , 120 , -30)$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -12 & 3 \\ 40 & 120 & -30 \end{vmatrix} = 120\hat{j} + 480\hat{k}$

16 ⓓ

17



Let the side length of the regular hexagon be ℓ cm.

$\because$ The two forces $(10\sqrt{3} , 10\sqrt{3})$ form a couple , its algebraic measure is M_1 where :

$M_1 = -10\sqrt{3} \times \ell\sqrt{3} = -30\,\ell$ newton.cm.

$\because$ The two forces (6 , 6) form a couple , its algebraic measure is M_2 where :

$M_2 = 6\,\ell$ newton.cm.

$\therefore M = M_1 + M_2 = -30\,\ell + 6\,\ell = -24\,\ell$ newton.cm.

$\therefore$ The norm of the moment of the resulting couple $= 24\,\ell$ newton.cm.

$\because AX = \frac{\sqrt{3}}{2}\ell$

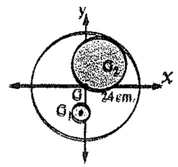
$\therefore$ For the system should be in equilibrium , then it need another couple of moment $= 24\,\ell$ acts with the previous system.

$\therefore$ The two forces act in the directions $\overrightarrow{EB}$ and $\overrightarrow{AO}$

$\therefore F \times \frac{\sqrt{3}}{2}\ell = 24\,\ell$ $\therefore F = 16\sqrt{3}$ newton.

$\therefore$ The two forces are $16\sqrt{3}$ and $16\sqrt{3}$ newton.

18



[a] Area of $G_1 : G_2 : G$

$= 16\pi : 144\pi : 576\pi$

$= 1 : 9 : 36$

, let mass of $G_1 = m$

, mass of $G_2 = 9$ m

, mass of G = 36 m

$\therefore G_1 = (-2 , -12) , G_2 = (6 , 10)$

Mass	− m	− 9 m	36 m
x	− 2	6	0
y	− 12	10	0

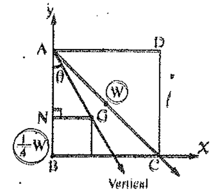
$\therefore x_G = \frac{-m \times -2 - 9m \times 6 + 0}{36m - m - 9m} = -2$

$, y_G = \frac{-m \times -12 - 9m \times 10 + 0}{36m - m - 9m} = -3$

$\therefore$ Centre of gravity $= (-2 , -3)$

[b]

m	w	$\frac{1}{4}$w
x	$\frac{1}{2}\ell$	0
y	$\frac{1}{2}\ell$	0



$\therefore x_G = \frac{w \times \frac{1}{2}\ell}{w + \frac{1}{4}w} = \frac{2}{5}\ell$

$, y_G = \frac{w \times \frac{1}{2}\ell}{w + \frac{1}{4}w} = \frac{2}{5}\ell$

$\therefore$ Centre of gravity $= \left(\frac{2}{5}\ell , \frac{2}{5}\ell\right)$

$\therefore GN = \frac{2}{5}\ell , AN = \frac{3}{5}\ell$

$\therefore \tan\theta = \frac{2}{5}\ell \div \frac{3}{5}\ell = \frac{2}{3}$

$\therefore m(\angle CAG) = 45° - m(\angle\theta)$

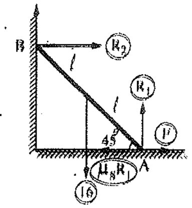
$\therefore \tan(\angle CAG) = \frac{1 - \frac{2}{3}}{1 + \frac{2}{3}} = \frac{1}{5}$

Model 3

1 ⓐ **2** ⓒ **3** ⓐ

4 ⓐ **5** ⓓ **6** ⓒ

7



$R_1 = 16$

$, F + R_2 = \mu_s R_1$

$\therefore F + R_2 = \frac{3}{4} \times R_1$

$\therefore F + R_2 = \frac{3}{4} \times 16$

$\therefore F + R_2 = 12$ kg.wt.

$M_A = 0$

$\therefore \ell \times 16 \times \cos 45° - 2\ell \times R_2 \sin 45° = 0$

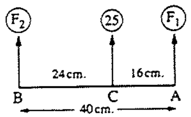
$\therefore R_2 = 8$ kg.wt.

$\therefore F = 12 - 8 = 4$ kg.wt.

8

[a] $M_A = F_2 \times 40 = 25 \times 16$

$\therefore F_2 = 10$ gm.wt.

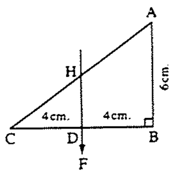


[b] $M_A = M_B$

$\therefore$ The line of action of $\overline{F} \mathbin{//} \overrightarrow{AB}$

$, \because M_B = -M_C$

$\therefore$ The line of action of $\overline{F}$ bisects $\overline{BC}$



$, \because M_B = F \times 4 = 60$

$\therefore F = 15$ newton.

, its line of action parallel $\overline{AB}$ and bisects $\overline{BC}$

i.e. acts in direction of $\overrightarrow{HD}$

---

9 ⓐ

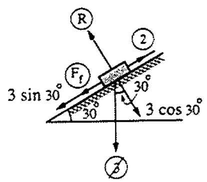
10

$\overline{M_O} = -300 \cos 30° \times 30 \sin 45° + 300 \sin 30° \times (30 \cos 45° + 40) = 3670.63$ newton.cm.

11

$\because w \sin \theta = 3 \sin 30° = 1.5$

$\therefore P > w \sin \theta$



$\therefore F_f$ acts in the direction of the line of the greatest slope downwards.

$\because$ The body is in equilibrium.

$\therefore P = F_f + w \sin \theta$

$\therefore 2 = F_f + 3 \times \frac{1}{2}$

$\therefore F_f = \frac{1}{2}$ newton downwards

$R = 3 \cos 30°$ $\quad \therefore R = \frac{3\sqrt{3}}{2}$

$\therefore \mu_s R = \frac{2}{3} \times \frac{3\sqrt{3}}{2} = \sqrt{3} = 1.732$ newton.

$\therefore \mu_s R > F_f$

$\therefore$ The force of static friction is not limiting

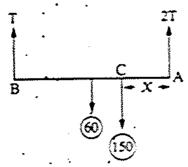
$\therefore$ The body is not about to move.

12

$2T + T = 150 + 60$

$T = 70$ newton.

$\overline{M_A} = 0$



$150 \times x + 60 \times 90 - T \times 180 = 0$

$\therefore 150\, x + 5400 - 70 \times 180 = 0$

$x = 48$ cm.

---

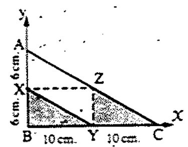
13 ⓐ **14** ⓒ **15** ⓐ

16

Area of Δ CYZ : area of Δ ABC = 1 : 4

, let mass of Δ CYZ = m

, mass of Δ ABC = 4 m



Mass	4 m	m	− m
x	$\frac{20}{3}$	$\frac{10}{3}$	$\frac{40}{3}$
y	4	2	2

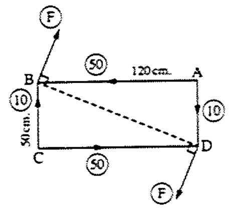
$\therefore x_G = \dfrac{4\,m \times \frac{20}{3} + m \times \frac{10}{3} - m \times \frac{40}{3}}{4\,m + m - m} = \frac{25}{6}$

$\therefore y_G = \dfrac{4\,m \times 4 + m \times 2 - m \times 2}{4\,m + m - m} = 4$

$, \tan \theta = 4 \div \frac{25}{6} = \frac{24}{25}$

17

[a]



$\because$ The two forces (10 , 10) form a couple , its algebraic measure is M_1 where :

$M_1 = -10 \times 120 = -1200$ newton.cm.

$\because$ The two forces (50 , 50) form a couple , its algebraic measure is M_2 where :

$M_2 = 50 \times 50 = 2500$ newton.cm.

$\because BD = \sqrt{(120)^2 + (50)^2} = 130$ cm.

, the two forces (F , F) form a couple , its algebraic measure is M_3 where :

$M_3 = -F \times 130 = -130\,F$ newton.cm.

In equilibrium :

$-1200 + 2500 - 130\,F = 0$

$\therefore F = 10$ newton.

[b] $\overline{OA} = \overline{A} - \overline{O} = (-1 , 1)$

$, \overline{OB} = \overline{B} - \overline{O} = (-2 , 3)$

$, \overline{OC} = \overline{C} - \overline{O} = (0 , 1)$

$\overline{M_o} = \overline{OA} \times \overline{F_1} + \overline{OB} \times \overline{F_2} + \overline{OC} \times \overline{F_3}$

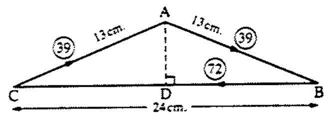
$= (-1 , 1) \times (2 , -4) + (-2 , 3) \times (1 , -3) + (0 , 1) \times (-3 , 7)$

$= 2\hat{k} + 3\hat{k} + 3\hat{k} = 8\hat{k}$ (1)

$, \overline{R} = \overline{F_1} + \overline{F_2} + \overline{F_3} = \overline{0}$ (2)

From (1) , (2) : The system is equivalent to couple its norm = 8 moment unit.

18



$AD = \sqrt{13^2 - 12^2} = 5$ cm.

$\because$ The forces taken in the same way around

$, \frac{39}{13} = \frac{72}{24} = \frac{39}{13} = 3$

$\therefore$ The system equivalent to couple

its norm = 2 × area of Δ ABC × 3

$= 2 \times \frac{1}{2} \times 24 \times 5 \times 3 = 360$ newton.cm.

Model 4

1 ⓒ	**2** ⓑ	**3** ⓑ	**4** ⓒ
5 ⓑ	**6** ⓐ	**7** ⓓ	**8** ⓑ

9

[a] $F = 84 + 36$

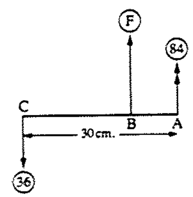
$= 120$ newton.

$M_A = 0$:

$F \times AB = 36 \times 30$

$120 \times AB = 1080$

$\therefore AB = 9$ cm. $\quad \therefore BC = 30 - 9 = 21$ cm.

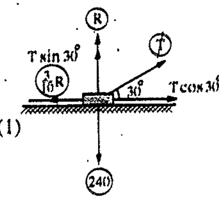


[b] $\because$ The body about to move

$\therefore T \cos 30° = \frac{3}{10} R$

$\therefore \frac{\sqrt{3}}{2} T \times \frac{10}{3} = R$

$\therefore R = \frac{5\sqrt{3}}{3} T$ (1)



$, R + T \sin 30° = 240$

$\therefore R + \frac{1}{2} T = 240$

By substitution from (1) :

$\therefore \left(\frac{5\sqrt{3}}{3} + \frac{1}{2}\right) T = 240 \quad \therefore \left(\frac{10\sqrt{3} + 3}{6}\right) T = 240$

$\therefore T = 70.86$ kg.wt.

---

10 ⓐ

11

$R_2 = 15$

$R_1 = \mu_s R_2 = 15\,\mu_s$

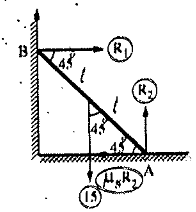
$M_A = 0 : \ell \times 15 \times \sin 45°$

$-2\,\ell \times R_1 \times \sin 45° = 0$

$R_1 = 7.5$ kg.wt.

$\therefore 7.5 = 15\,\mu_s$

$\therefore \mu_s = \frac{1}{2}$



12

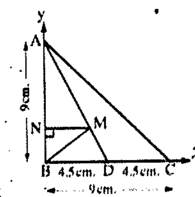
[a] $\because M = \left(\frac{9+0+0}{3} , \frac{9+0+0}{3}\right) = (3 , 3)$

$\therefore MN = 3$ cm.

The area of Δ ABM : Area of Δ ABC

$= \frac{1}{2} \times 9 \times 3 :$

$\frac{1}{2} \times 9 \times 9 = 1 : 3$



Let the mass of Δ ABM = k

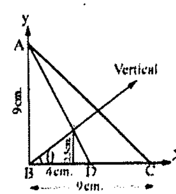
The mass of Δ ABC = 3 k

The gravity centre of Δ AMB

$= \left(\frac{0+0+3}{3} , \frac{9+0+3}{3}\right)$

$= (1 , 4)$

Mass	− k	3 k
x	1	3
y	4	3



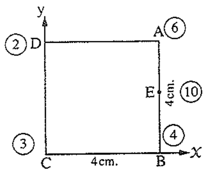
$\therefore X_G = \dfrac{-k \times 1 + 3k \times 3}{3k - k} = 4$

$y_G = \dfrac{-k \times 4 + 3k \times 3}{3k - k} = 2.5$

, $\tan\theta = \dfrac{2.5}{4} = \dfrac{5}{8}$

[b] Let the two perpendicular

direction $\overrightarrow{CX}$, $\overrightarrow{Cy}$



	C	B	A	D	E
k	3	4	6	2	10
X	0	4	4	0	4
y	0	0	4	4	2

$X_G = \dfrac{3 \times 0 + 4 \times 4 + 6 \times 4 + 2 \times 0 + 10 \times 4}{3 + 4 + 6 + 2 + 10} = 3.2$

$y_G = \dfrac{3 \times 0 + 4 \times 0 + 6 \times 4 + 2 \times 4 + 10 \times 2}{3 + 4 + 6 + 2 + 10} = 2.08$

i.e. The centre of gravity at a distance 3.2 cm. from $\overline{CD}$

, 2.08 cm. from $\overline{CB}$

13

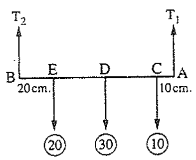
$T_1 + T_2 = 60$

$M_B = 0 : 10 \times 50 + 30 \times 30$

$+ 20 \times 20 = T_1 \times 60$

$T_1 = 30$ newton

, $T_2 = 30$ newton.



14

$\because$ The rod is in equilibrium

$\therefore 6 \times 40 = F \times 20 \times \sin 30°$

$\therefore F = 24$ newton.

15

$\because$ The body is about to move under effect of its weight only if it is placed on a rough inclined plane

, makes angle its tangent $\frac{1}{4}$

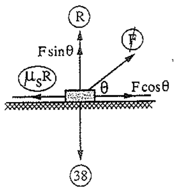
$\therefore \mu_s = \frac{1}{4}$

$R + F \sin\theta = 38$

$\therefore R + \frac{3}{5} F = 38$ (1)

, $F \cos\theta = \mu_s R$

$\frac{4}{5} F = \frac{1}{4} R$

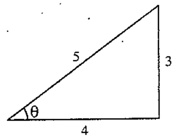


$\therefore R = \frac{16}{5} F$ (2)

By substitution from (2) in (1) :

$\frac{16}{5} F + \frac{3}{5} F = 38$

$\therefore F = 10$ kg.wt.



16 ⓑ

17

From the figure :

$A = (0 , 0 , 10)$, $D = (6 , 8 , 0)$

$\therefore \overrightarrow{AD} = \overrightarrow{D} - \overrightarrow{A} = (6 , 8 , -10)$

$\therefore \overrightarrow{F} = F \times \dfrac{\overrightarrow{AD}}{\|\overrightarrow{AD}\|} = 20\sqrt{2} \times \dfrac{(6 , 8 , -10)}{\sqrt{(6)^2 + (8)^2 + (-10)^2}}$

$= (12 , 16 , -20)$

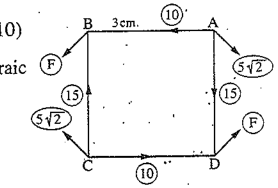
$\therefore \overrightarrow{M_O} = \overrightarrow{OA} \times \overrightarrow{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 10 \\ 12 & 16 & -20 \end{vmatrix} = -160\hat{i} + 120\hat{j}$

18

The two forces (10 , 10) form a couple of algebraic measure (M_1)

$M_1 = 10 \times 3$

$= 30$ newton.cm.



and the two forces (15 , 15) form a couple of algebraic measure (M_2)

$M_2 = -15 \times 3 = -45$ newton.cm.

$\because AC = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

The two forces $(5\sqrt{2} , 5\sqrt{2})$ form a couple of algebraic measure (M_3)

$M_3 = -5\sqrt{2} \times 3\sqrt{2} = -30$ newton.cm.

$M = M_1 + M_2 + M_3 = 30 + (-45) + (-30)$

$= -45$ newton.cm.

$\therefore BD = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$\therefore$ The two forces (F , F) form a couple of algebraic measure $M_4 = F \times 3\sqrt{2} = 3\sqrt{2}\,F$ newton.cm.

, $\because M = -M_4$

$\therefore 3\sqrt{2}\,F = 45$

$\therefore F = \dfrac{15\sqrt{2}}{2}$ newton.

Model 5

1 ⓒ **2** ⓒ

3

[a] $R = F_1 - F_2$

$40 = 60 - F$

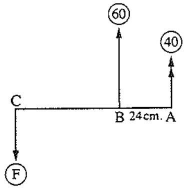
$\therefore F = 20$ newton.

$M_A = 0 \quad \therefore 60 \times 24 = F \times AC$

$1440 = 20 \times AC$

$AC = 72$

$\therefore BC = 72 - 24 = 48$ cm.



[b] $M_x = y F_z - Z F_y$

$\therefore -1 = 1 \times -2 - 1 \times m$

$\therefore -1 = -2 - m$ $\quad \therefore m = -1$

, $M_y = Z F_x - X F_z$

$\therefore -8 = 1 \times k - 3 \times -2$

$\therefore -8 = k + 6$ $\quad \therefore k = -14$

4 ⓓ **5** ⓑ **6** ⓑ **7** ⓐ

8

$2T + T = 6w$

$T = 2w$ (1)

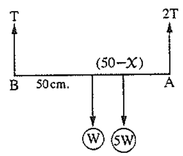
$M_B = 0 :$

$-w \times 50 - 5w \times (100 - X)$

$+ 2T \times 100 = 0$

$\therefore -50w - 500w + 5wX + 200T = 0$

$-550w + 5wX + 200T = 0$ (2)



By substitution from (1) in (2) :

$-550w + 5wX + 200 \times 2w = 0$ ($\div$ w)

$5X = 150 \quad \therefore X = 30$

$\therefore$ The weight (5 w) hangs at distance 30 cm. from one end to make tension near to it double the other tension.

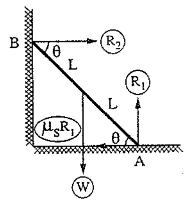
9

Let the rod length be 2 L and its weight w

$R_1 = w , \mu_s R_1 = R_2$

$\therefore \frac{1}{4} R_1 = R_2$

$\therefore R_2 = \frac{1}{4} w$



$M_A = 0 :$

$w \times L \cos\theta - R_2 \times 2L \times \sin\theta = 0$

$\therefore w \cos\theta - 2 \times \frac{1}{4} w \sin\theta = 0$

$\cos\theta = \frac{1}{2} \sin\theta$

$\therefore \tan\theta = 2$

10

The rectangle area $= 24 \times 18 = 432$

and its centre (12 , 9)

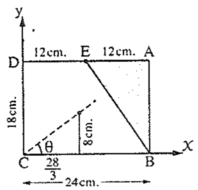
The area of triangle ABE

$= \frac{1}{2} \times 12 \times 18 = 108$

and its centre

$\left(\dfrac{24 + 24 + 12}{3} , \dfrac{0 + 18 + 18}{3}\right)$

$= (20 , 12)$



Mass	432	− 108
X	12	20
y	9	12

$\therefore X_G = \dfrac{432 \times 12 - 108 \times 20}{432 - 108} = \dfrac{28}{3}$

$\therefore y_G = \dfrac{432 \times 9 - 108 \times 12}{432 - 108} = 8$

$\therefore \tan\theta = \dfrac{8}{\left(\frac{28}{3}\right)} = \dfrac{6}{7}$

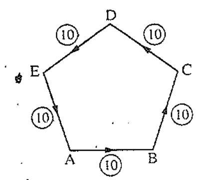
11 ⓒ **12** ⓒ

13

$\because$ The forces in the same way round.

, $\because \dfrac{10}{15} = \dfrac{10}{15} = \dfrac{10}{15}$

$= \dfrac{10}{15} = \dfrac{10}{15} = \dfrac{2}{3}$



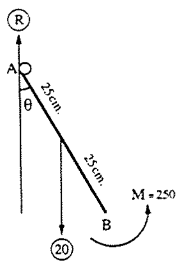
$\therefore$ The system is equivalent to a couple , its algebraic measure $= 2 \times$ area of the pentagon $\times \frac{2}{3}$

$= 2 \times \frac{5}{4} \times (15)^2 \times \left(\cot \frac{180°}{5}\right) \times \frac{2}{3}$

$\simeq 516.14$ kg.wt.cm.

$\therefore$ The norm of the moment of the couple $\simeq 516.14$ kg.wt.cm.

14



∵ The rod is in equilibrium under effect of two couples

∴ The two forces (20 , R) form a couple whose algebraic measure = – 250

∴ – 20 × 25 × sin θ = – 250

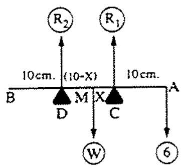
$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30°$ or 150°

15

Let the weight of the rod be w kg.wt. acts at point M where CM = X

* When the weight 6 kg.wt. is suspended at A :



∵ The rod is about to overturn about C

∴ R_2 = zero

∴ The rod is in equilibrium under effect of 3 forces

R_1 , w , 6 kg.wt.

∴ 6 × 10 = w × X

wX = 60 (1)

* When the weight 9 kg.wt. is suspendend at B :

∵ The rod is about to overturn about D

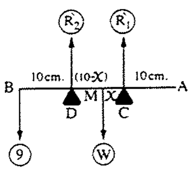
∴ $\bar{R}_1$ = zero

∴ The rod is in equilibrium under effect of 3 forces

$\bar{R}_2$, w , 9 kg.wt.

∴ w × (10 – X) = 9 × 10

10 w – wX = 90 (2)



By substituting from (1) in (2) :

10 w – 60 = 90

∴ w = 15

By substituting in (1) : 15 X = 60

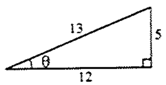
∴ X = 4 cm.

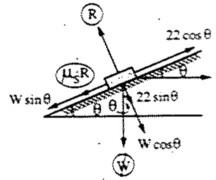
∴ The weight acts at a point for from A = 10 + 4 = 14 cm.

16 ⓒ **17** ⓒ

18

[a]





∵ The body is about to move upwards

∴ R = 22 sin θ + w cos θ

∴ $R = 22 \times \frac{5}{13} + \frac{12}{13}$ w. (1)

, 22 cos θ = μ_s R + w sin θ

By substituting in (1) :

$\therefore 22 \times \frac{12}{13} = \frac{1}{2} \times \left(\frac{110}{13} + \frac{12}{13} w\right) + w \times \frac{5}{13}$ (multiply by 13)

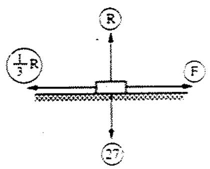
∴ 264 = 55 + 6 w + 5 w

∴ 11 w = 209

∴ w = 19 newton.

[b] ∵ The friction is limiting

∴ The equations of equilibrium are :



$F = \frac{1}{3} R$

, R = 27

By substituting from (2) in (1) :

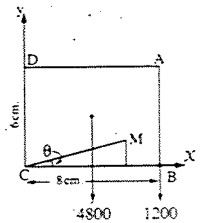
$\therefore F = \frac{1}{3} \times 27 = 9$ kg.wt.

Model 6

1 ⓐ **2** ⓑ **3** ⓑ **4** ⓒ

5

k	4800	1200
X	4	8
y	3	0



$X_G = \frac{4800 \times 4 + 1200 \times 8}{4800 + 1200}$

= 4.8 cm.

$y_G = \frac{4800 \times 3 + 1200 \times 0}{4800 + 1200} = 2.4$ cm.

$\tan\theta = \frac{2.4}{4.8} = \frac{1}{2}$

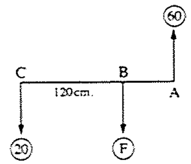
6 ⓑ **7** ⓒ

8

[a] ∵ R = F – 60

∴ 20 = F – 60

F = 80 kg.wt.



∵ $M_C = 0$:

– F × 120 + 60 × AC = 0

∴ – 80 × 120 + 60 × AC = 0

∴ AC = 160

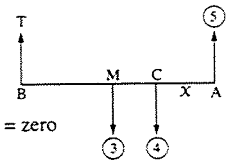
∴ The rod length = 160 – 120 = 40 cm.

[b] From the equilibrium condition

∴ 5 + T = 7

∴ T = 2

, M_A = zero



∴ 4 X + 3 × 40 – 2 × 80 = zero

∴ X = 10

∴ The weight should be suspended at a distance not less than 10 cm. away from any of the two ends.

Another solution :

∵ $T_1 + T_2 = 7$

, ∵ each of the two tensions can not bear tension more that 5 kg.wt.

∴ The least tension in the other string

= 7 – 5 = 2 kg.wt.

∴ 2 ≤ tension in any string ≤ 5

∵ $M_A = 0$ ∴ 4 X + 3 × 40 = T × 80

, ∵ $T = \frac{4X + 120}{80} = \frac{X + 30}{20}$

$\therefore 2 \le \frac{X + 30}{20} \le 5$

∴ 10 ≤ X ≤ 70

∴ The weight can be suspended at a distance between 10 cm. , 70 cm. from A or exactly at 10 cm. , 70 cm. from A

9

∵ The rod is about to move away from the wall

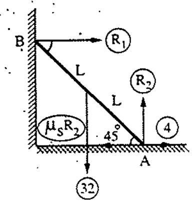
∴ The friction acts in the wall direction

the equations of equilibrium

R_2 = 32 kg.wt.

$R_1 + 4 = \mu_s R_2$

$R_1 = 32 \mu_s - 4$ (1)



$M_A = 0$:

∴ L × 32 × cos 45° – 2 L × R_1 × sin 45° = 0 (2)

by substituting from (1) in (2) :

$\therefore 32\left(\frac{1}{\sqrt{2}}\right) - 2(32\mu_s - 4) \times \left(\frac{1}{\sqrt{2}}\right) = 0$

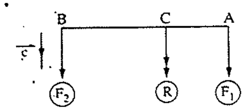
$32 - 64\mu_s + 8 = 0$, $\therefore \mu_s = \frac{5}{8}$

10 ⓑ **11** ⓐ

12

• Before moving :

$F_1 \times AC = F_2 \times BC$ (1)

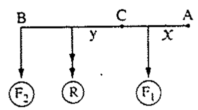


• After moving :

$F_1 (AC - X + y) = F_2 (BC - y)$ (2)

$\therefore F_1 AC - F_1 X + F_1 y$

$= F_2 BC - F_2 y$



By substituting from (1) in (2) :

$\therefore -F_1 X + F_1 y = -F_2 y$

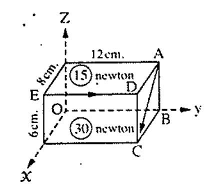
$\therefore (F_1 + F_2) y = F_1 X$

$\therefore y = \frac{F_1}{F + F_2} \times X$

13 ⓐ

14

From the figure



$\overrightarrow{ED} = \overrightarrow{D} - \overrightarrow{E}$

= (8 , 12 , 6) – (8 , 0 , 6)

= (0 , 12 , 0)

$\therefore \overrightarrow{F_1} = 15 \times \frac{\overrightarrow{ED}}{\|\overrightarrow{ED}\|}$

$= 15 \times \frac{(0, 12, 0)}{\sqrt{(0)^2 + (12)^2 + (0)^2}} = (0, 15, 0)$

$, \overrightarrow{AC} = \overrightarrow{C} - \overrightarrow{A} = (8, 12, 0) - (0, 12, 6)$

= (8 , 0 , –6)

$\therefore \overrightarrow{F_2} = 30 \times \frac{\overrightarrow{AC}}{\| \overrightarrow{AC} \|}$

$= 30 \times \frac{(8 , 0 , -6)}{\sqrt{(8)^2 + (0)^2 + (-8)^2}} = (24 , 0 , -18)$

$\therefore \overrightarrow{M_O} = \overrightarrow{OA} \times \overrightarrow{F_2} + \overrightarrow{OE} \times \overrightarrow{F_1}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 12 & 6 \\ 24 & 0 & -18 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & 6 \\ 0 & 15 & 0 \end{vmatrix}$

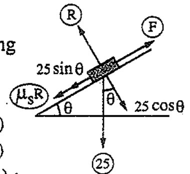
$= -216 \hat{i} + 144 \hat{j} - 288 \hat{k} - 90 \hat{i} + 120 \hat{k}$

$= -306 \hat{i} + 144 \hat{j} - 168 \hat{k}$

15 ⓐ

16

[a] In case the body is about to move up the friction is limiting and acts down (F = 15)



The equilibrium equations :

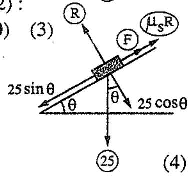
$R = 25 \cos \theta$ (1)

$F = 25 \sin \theta + \mu_s R$ (2)

By substituting from (1) in (2) :

$15 = 25 \sin \theta + \mu_s \times (25 \cos \theta)$ (3)

* In case the body is about to move down the friction is limiting and acts up (F = 10)



The equilibrium equations :

$R = 25 \cos \theta$ (4)

$\mu_s R + F = 25 \sin \theta$ (5)

By substituting from (4) in (5) :

$\mu_s (25 \cos \theta) + 10 = 25 \sin \theta$ (6)

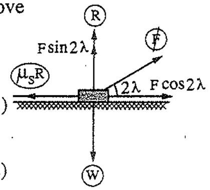
By adding (3) , (6) :

$25 + 25 \cos \theta \, \mu_s = 50 \sin \theta + 25 \cos \theta \, \mu_s$

$\frac{1}{2} = \sin \theta$ $\therefore \theta = 30°$

∴ The plane inclined to the horizontal by 30°

[b] ∵ The body is about to move



$\therefore F \cos 2\lambda = \mu_s R$

$, \because \mu_s = \tan \lambda$

$\therefore F \cos 2\lambda = R \tan \alpha$ (1)

$, R + F \sin 2\lambda = W$

$\therefore R = W - F \sin 2\lambda$ (2)

From (1) , (2) :

$\therefore F \cos 2\lambda = \tan \lambda \, [W - F \sin 2\lambda]$

$\therefore F \cos 2\lambda = \frac{\sin \lambda}{\cos \lambda} [W - F \sin 2\lambda]$

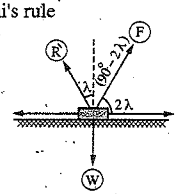
$\therefore F \cos 2\lambda \cos \lambda = W \sin \lambda - F \sin \lambda \sin 2\lambda$

$\therefore F [\cos 2\lambda \cos \lambda + \sin 2\lambda \sin \lambda] = W \sin \lambda$

$\therefore F \cos \lambda = W \sin \lambda$

$\therefore F = \frac{W \sin \lambda}{\cos \lambda}$ $\therefore F = W \tan \lambda$

Another solution by using lami's rule



$\frac{F}{\sin (180° - \lambda)} = \frac{W}{\sin (90° - \lambda)}$

$= \frac{R}{\sin (90° + 2\lambda)}$

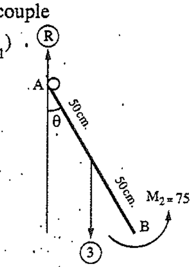
$\therefore \frac{F}{\sin \lambda} = \frac{W}{\cos \lambda}$

$\therefore F = \frac{W \sin \lambda}{\cos \lambda} = W \tan \lambda$

17

∵ The rod is in equilibrium under effect of two couples M_1 , M_2

∴ The two forces (R , 3) form couple whose algebraic measure (M_1)



∴ R = 3 kg.wt.

$, M_1 = -3 \times 50 \times \sin \theta$

$, M_2 = 75$

$\because M_1 = -M_2$

$\therefore -3 \times 50 \times \sin \theta = -75$

$\sin \theta = \frac{1}{2}$

θ = 30° or 150°

18

In Δ ABX : $AX = \sqrt{9^2 + 12^2} = 15$ cm.

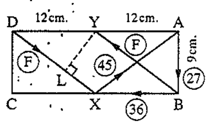
∵ The forces (27 , 36 , 45) act in one cyclic order

$, \frac{27}{9} = \frac{36}{12} = \frac{45}{15} = 3$

∴ The forces (27 , 36 , 45) form a couple whose algebraic measure (M_1)

$M_1 = -2 \times$ the area of Δ ABX × 3

$= -2 \times \frac{1}{2} \times 12 \times 9 \times 3 = -324$ newton. cm.



In Δ YDL : $\overline{YL} \perp \overline{XD}$

m (∠ YDL) = m (∠ DXC)

∴ YL = 12 sin (∠ YDL)

= 12 sin (∠ DXC)

$= 12 \times \frac{9}{15} = 7.2$ cm.

Let the two forces (F , F) act in direction $\overrightarrow{BY} , \overrightarrow{DX}$ and form a couple whose algebraic measure (M_2)

$M_2 = F \times 7.2$

∵ The system is in equilibrium

$\therefore M_1 + M_2 = 0$

$\therefore -324 + F \times 7.2 = 0$

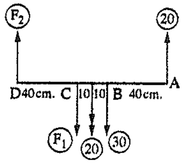
∴ F = 45 newton

∴ The two forces are (45 , 45) newton

Model 7

1 ⓒ **2** ⓑ **3** ⓒ **4** ⓐ **5** ⓒ

6



$20 = (30 + F_1) - (20 + F_2)$

$\therefore F_1 - F_2 = 10$ (1)

∵ The moments sum about D = the resultant moment about D

$\therefore 20 \times 100 - 30 \times 60 - F_1 \times 40$

$= -20 \times 50$

$\therefore F_1 = 30$ newton

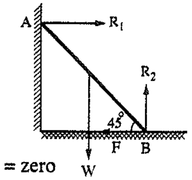
, by substituting in (1) :

$F_2 = 20$ newton.

7 ⓓ

8

The equilibrium equations :



$R_1 = F$

$R_2 = w$

$, M_B = zero$

$\therefore w \times \frac{L}{2} \cos 45° - R_1 \, L \sin 45° = zero$

$\frac{w}{2} = R_1$ $\therefore F = \frac{1}{2} w$

$, \because \mu_s R_2 = \mu_s w$

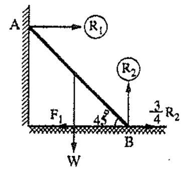
To make the rod in equilibrium , it must be $F_s \geq F$

$\mu_s w \geq \frac{1}{2} w$

$\therefore \mu_s \geq \frac{1}{2}$

i.e. the coefficient of friction must be not less than $\frac{1}{2}$ and assuming that the force makes the rod about to move towards the wall $= F_1$

∴ The equilibrium equations are



$R_1 + \frac{3}{4} R_2 = F_1$

$R_2 = w$

$\therefore R_1 + \frac{3}{4} w = F_1$

$, M_B = zero$

$\therefore w \times \frac{L}{2} \cos 45° - R_1 \, L \sin 45° = zero$

$\therefore R_1 = \frac{w}{2}$

$\therefore F_1 = \frac{1}{2} w + \frac{3}{4} w = \frac{5}{4} w$

9 ⓑ

10

$\because M_A = \pm 200$ newton.cm.

$\therefore -F \cos 30° \times 40 - F \sin 30° \times 5 = -200$

$\therefore F (40 \cos 30° + 5 \sin 30°) = 200$

∴ F = 5.38 newton.

11 ⓑ

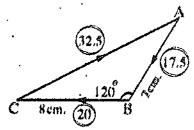
12

In Δ ABC

$AC = \sqrt{7^2 + 8^2 - 2 \times 7 \times 8 \cos 120°} = 13$ cm.

∵ The forces act in one cyclic order

$, \frac{17.5}{7} = \frac{20}{8} = \frac{32.5}{13} = \frac{5}{2}$



∴ The system equivalent to a couple whose algebraic measure

$= -2 \times$ the area of triangle $\times \frac{5}{2}$

$= -2 \times \frac{1}{2} \times 8 \times 7 \times \sin 120° \times \frac{5}{2}$

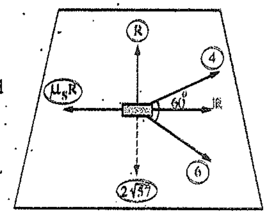
$= -70\sqrt{3}$ newton. cm.

∴ The magnitude of the moment $= 70\sqrt{3}$ newton. cm.

13

[a] $\mathbb{R} = \sqrt{4^2 + 6^2 + 2 \times 4 \times 6 \cos 60°}$

$= 2\sqrt{19}$ kg.wt.



, ∵ the body is in equilibrium and about to move

$\therefore F_s = 2\sqrt{19}$ kg.wt.

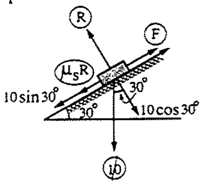
$, \because R = 2\sqrt{57}$

∴ The coefficient of friction $= \frac{F_s}{R} = \frac{2\sqrt{19}}{2\sqrt{57}} = \frac{\sqrt{3}}{3}$

[b] ∵ The body about to slide under effect of his weight

∴ $\mu_s = \tan 30° = \frac{1}{\sqrt{3}}$

∵ The body about to move upward



∴ $F = \mu_s R + w \sin\theta$

∴ $F = \frac{1}{\sqrt{3}} R + 10 \sin 30°$

∴ $F = \frac{1}{\sqrt{3}} R + 5$ (1)

, $R = 10 \times \cos 30° = 5\sqrt{3}$ (2)

By substituting in (1) :

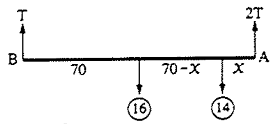
∴ $F = \frac{1}{\sqrt{3}} \times 5\sqrt{3} + 5 = 5 + 5 = 10$ kg.wt.

14 ⓑ

15

From the equilibrium conditions :

$3T = 14 + 16 = 30$



∴ T = 10 newton

, $M_A = 0$:

$X \times 14 + 70 \times 16 - 140 \times T = 0$

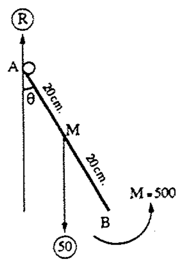
$14X + 1120 - 140 \times 10 = 0$

∴ $X = 20$ cm.

∴ The weight is suspended at a distance 20 cm. from the end A

16

∵ The rod is in equilibrium under effect of two couples



∴ The two forces (50 , R) form a couple whose algebraic measure = − 500 kg.wt.cm.

∴ R = 50 vertically upwards

∴ $-50 \times 20 \times \sin\theta = -500$

$\sin\theta = \frac{1}{2}$

∴ θ = 30° or 150°

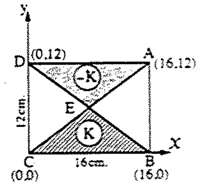
∴ The rod inclined to the vertical downward at an angle of measure 30° or 150°

17 ⓐ

18

[a] Let the two perpendicular directions be $\overrightarrow{CB}$, $\overrightarrow{CD}$

k	4 k	k	− k
x	8	8	8
y	6	2	10



$x_G = \frac{4k \times 8 + k \times 8 - k \times 8}{4k + k - k}$

$= 8$

$y_G = \frac{4k \times 6 + k \times 2 - k \times 10}{4k + k - k} = \frac{16}{4} = 4$

∴ The centre of gravity of the lamina = (8 , 4)

[b] $x_G = \frac{1 \times 0 + 1 \times 3 + 2 \times 3}{1 + 1 + 2} = \frac{9}{4}$

$y_G = \frac{1 \times 0 + 1 \times 0 + 2 \times 4}{1 + 1 + 2} = 2$

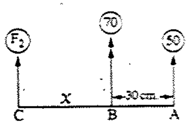
∴ The centre of gravity of the system is $\left(\frac{9}{4} , 2\right)$

Model 8

1 ⓒ

2

$R = F_1 + F_2$



$70 = 50 + F_2$

∴ $F_2 = 20$ newton

The moment of the forces about C = the moment of the resultant about C

$50 \times (30 + X) = 70 \times X$

∴ $1500 + 50X = 70X$

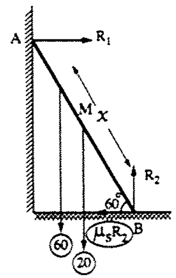
∴ $X = 75$

∴ The force $F_2 = 20$ newton and acts in the same direction as F_1 and at a distance 75 + 30 = 105 cm. from it.

3 ⓒ **4** ⓒ **5** ⓒ **6** ⓐ **7** ⓐ

8

Let the girl ascends X



, the ladder is about to slide

the equilibrium equations

$R_1 = \frac{1}{2\sqrt{3}} R_2$

, $R_2 = 80$

∴ $R_1 = \frac{1}{2\sqrt{3}} \times 80 = \frac{40\sqrt{3}}{3}$

, $M_B = $ zero

∴ $20 \times \frac{L}{2} \cos 60° + 60X \cos 60° - R_1 \times L \sin 60° = $ zero

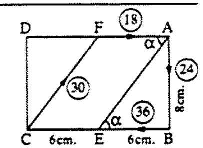
∴ $5L + 30X = \frac{40\sqrt{3}}{3} L \times \frac{\sqrt{3}}{2}$

∴ $30X = 20L - 5L = 15L$

∴ $X = \frac{1}{2} L$

i.e. the greatest distance that the girl can ascend equals half the ladder length.

9



From the figure :

$AE = FC = \sqrt{8^2 + 6^2} = 10$ cm.

∵ The forces act in one cyclic order in the quadrilateral ABCF

, $\frac{24}{8} = \frac{36}{12} = \frac{30}{10} = \frac{18}{6} = 3$

∴ The forces form a couple whose algebraic measure M_1 where

$M_1 = -2 \times$ the area of the trapezium ABCD × 3

∴ $M_1 = -2 \times \frac{1}{2} (12 + 6) \times 8 \times 3 = -432$ newton.cm.

Let the two forces (F , F) act in direction of $\overrightarrow{EA}$, $\overrightarrow{FC}$ form a couple whose algebraic measure M_2

∴ $M_1 = -M_2$

∴ $F \times 6 \times \sin\alpha = 423$

∴ $F \times 6 \times \frac{8}{10} = 423$

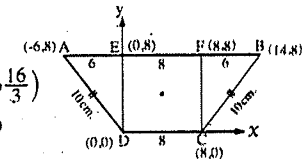
∴ F = 90 newton.

10 ⓐ

11

∵ The area of the square CDEF

= 8 × 8 = 64 and its centre (4 , 4)



the area of Δ CBF

$= \frac{1}{2} \times 6 \times 8 = 24$

and its centre $\left(10 , \frac{16}{3}\right)$

The area of Δ AED

$= \frac{1}{2} \times 6 \times 8 = 24$

and its centre $\left(-2 , \frac{16}{3}\right)$

The mass	64	24	24
x	4	10	− 2
y	4	$\frac{16}{3}$	$\frac{16}{3}$

$x_G = \frac{64 \times 4 + 24 \times 10 + 24 \times -2}{64 + 24 + 24} = 4$.

$y_G = \frac{64 \times 4 + 24 \times \frac{16}{3} + 24 \times \frac{16}{3}}{64 + 24 + 24} = \frac{32}{7}$

and when the weight ($\grave{k}$) suspended at A and the lamina is about to rotate

∴ The centre of the lamina after adding $\grave{k}$ must lies on the line perpendicular to $\overline{CD}$ and passes through D

i.e. $\grave{x}_G = $ zero

Let the lamina mass be k

Mass	k	$\grave{k}$
$\grave{x}$	4	− 6
y	$\frac{32}{7}$	8

$\grave{x}_G = \frac{k \times 4 - 6\grave{k}}{k + \grave{k}} = 0$

∴ $4k = 6\grave{k}$

∴ $\grave{k} = \frac{2}{3} k$

i.e. the greatest weight can be suspended at A = $\frac{2}{3}$ the lamina weight

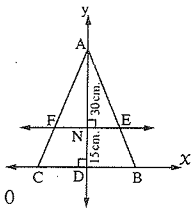
12

[a] $\frac{\text{Area of } (\Delta AEF)}{\text{Area of } (\Delta ABC)} = \left(\frac{AN}{AD}\right)^2 = \frac{4}{9}$

∴ The mass of triangular lamina AEF = 4 m

, the mass of triangular lamina ABC = 9 m

Mass	– 4 m	9 m
x	0	0
y	25	15



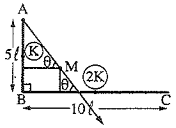
$\therefore x_G = \frac{-4\,m \times 0 + 9\,m \times 0}{-4\,m + 9\,m} = 0$

$, y_G = \frac{-4\,m \times 25 + 9\,m \times 15}{-4\,m + 9\,m} = 7$

∴ The centre of gravity of the quadrilateral EBCF lies on $\overline{AD}$ and at a distance 7 cm. from D

[b]

The mass	2 k	k
x	5 ℓ	0
y	0	2.5 ℓ



$x_G = \frac{2\,k \times 5\,\ell}{3\,k} = \frac{10}{3}\,\ell$

$, y_G = \frac{k \times 2.5\,\ell}{3\,k} = \frac{5}{6}\,\ell$

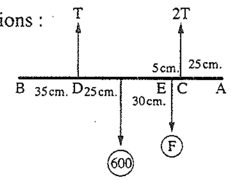
∴ The centre of gravity = $\left(\frac{10}{3}\,\ell , \frac{5}{6}\,\ell\right)$

$\therefore \tan\theta = \frac{5\,\ell - \frac{5}{6}\,\ell}{\frac{10}{3}\,\ell} = \frac{5}{4}$

∴ $\overline{BC}$ inclined to the horizontal at an angle whose tangent = $\frac{4}{5}$

13

From the equilibrium equations :



3 T = F + 600 (1)

, ∵ the algebraic measure of sum of moments about E = zero

∴ 2 T × 5 + 600 × 30 – T × 55 = zero

∴ 10 T + 18000 – 55 T = 0

∴ T = 400

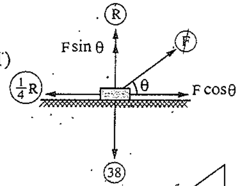
By substituting in (1) :

3 × 400 = F + 600

∴ F = 600

14

[a] From the equilibrium equations :



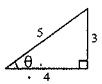
R + F sin θ = 38

$\therefore R + \frac{3}{5} F = 38$ (1)

$, F \cos\theta = \frac{1}{4} R$

$\therefore F \times \frac{4}{5} = \frac{1}{4} R$

$\therefore F = \frac{5}{16} R$ (2)



By substituting from (2) in (1) :

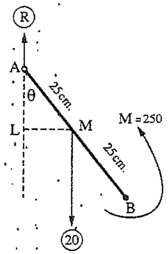
$\therefore R + \frac{3}{5}\left(\frac{5}{16} R\right) = 38$ ∴ R = 32 newton

$F_s = \frac{1}{4} \times 32 = 8$ newton.

$\therefore \bar{R} = \sqrt{(32)^2 + (8)^2} = 8\sqrt{17}$ newton.

[b] ∵ The rod is in equilibrium under effect of two couples

∴ The two forces (20 , R) form a couple whose algebraic measure = – 250 newton.cm.



∴ R = 20 newton vertically upwards

, – 20 × ML = – 250

∴ ML = 12.5 cm.

$\therefore \sin\theta = \frac{12.5}{25} = \frac{1}{2}$

∴ θ = 30° or 150°

∴ The rod inclined to the vertical downwards at an angle of measure 30° or 150°

15 ⓒ **16** ⓑ

17

From the figure :

A = (0 , 0 , 6) , B = (0 , 4 , 0) , C = (4 , –3 , 0)

$\overrightarrow{AB} = \vec{B} - \vec{A} = (0 , 4 , -6)$

$\overrightarrow{AC} = \vec{C} - \vec{A} = (4 , -3 , -6)$

$\therefore \vec{F_1} = F_1 \times \frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|} = 6\sqrt{13} \times \frac{(0 , 4 , -6)}{\sqrt{(0)^2 + (4)^2 + (-6)^2}}$

= (0 , 12 , – 18)

$, \vec{F_2} = F_2 \times \frac{\overrightarrow{AC}}{\|\overrightarrow{AC}\|} = \sqrt{61} \times \frac{(4 , -3 , -6)}{\sqrt{(4)^2 + (-3)^2 + (-6)^2}}$

= (4 , – 3 , – 6)

The sum of the moments about "O"

$= M_1 + M_2 = \overrightarrow{OA} \times \vec{F_1} + \overrightarrow{OC} \times \vec{F_2}$

= (0 , 0 , 6) × (0 , 12 , – 18) + (4 , – 3 , 0) × (4 , – 3 , – 6)

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 6 \\ 0 & 12 & -18 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 0 \\ 4 & -3 & -6 \end{vmatrix}$

$= -72\hat{i} + 18\hat{i} + 24\hat{j} = -54\hat{i} + 24\hat{j}$

18 ⓐ

Model 9

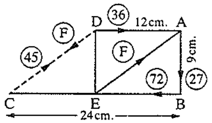
1 ⓑ **2** ⓑ **3** ⓐ **4** ⓐ

5

From the figure :

$CD = \sqrt{9^2 + 12^2} = 15$ cm.

∵ The forces acting on the same way round in trapezium ABCD



$, \frac{-27°}{9} = \frac{72}{24} = \frac{45}{15} = \frac{36}{12} = 3$

∴ The forces is equivalent to a couple

, its algebraic measure

= – 2 × area of trapezium × 3

$\therefore \mu_1 = -2 \times \frac{1}{2} \times (24 + 12) \times 9 \times 3 = -972$ newton.cm.

Let the two forces (F , F) acting upon $\overrightarrow{EA}$, $\overrightarrow{DC}$

and form a couple its algebraic measure

$\mu_2 = F \times 12 \times \sin(\angle EAD) = F \times 12 \times \frac{9}{15} = \frac{36}{5} F$

∵ The system is equilibrium

$\therefore M_1 + M_2 = 0$ $\therefore -972 + \frac{36}{5} F = 0$

∴ F = 135 newton.

6 ⓒ **7** ⓑ **8** ⓒ

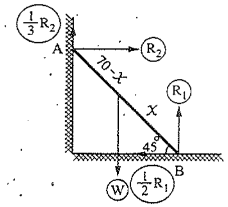
9

Let weight of rod "w" and at distant x from the point (B)

∴ Equilibrium equations are :

$R_1 + \frac{1}{3} R_2 = w$ (1)

$, \frac{1}{2} R_1 = R_2$ (2)



From (1) , (2) :

$R_1 + \frac{1}{3} \times \frac{1}{2} R_1 = w$

$\therefore \frac{7}{6} R_1 = w$

$\therefore R_1 = \frac{6}{7} w$ (3)

$, R_2 = \frac{3}{7} w$ (4)

$, \mu_B = 0$

$\therefore x \times w \cos 45° - 70\, R_2 \times \sin 45° - 70 \times \frac{1}{3} R_2 \cos 45° = 0$

(divided by cos 45°)

$\therefore x \times w - 70 \times \frac{3}{7} w - 70 \times \frac{1}{3} \times \frac{3}{7} w = 0$

divided by w

∴ x = 40 cm.

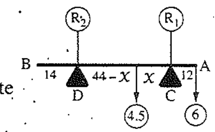
∴ Centre of gravity of the rod distant 40 cm. from the point (B)

10 ⓒ

11

[a] Let the distance between centre of gravity of the rod and point (C) = x

∵ The rod is about to rotate about C



∴ R_2 = 0

∴ 6 × 12 = 4.5 × x

x = 16 cm.

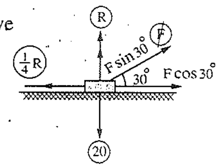
∴ Centre of gravity of the rod at distance 16 cm. from the point (C)

[b] ∵ The body about to move

$\therefore F \cos 30° = \frac{1}{4} R$

$\therefore \frac{\sqrt{3}}{2} F = \frac{1}{4} R$

$\therefore R = 2\sqrt{3} F$ (1)



, R + F sin 30° = 20

Substitute from (1) : ∴ $2\sqrt{3}\,F + \frac{1}{2}\,F = 20$

∴ $F = \dfrac{20}{2\sqrt{3} + \frac{1}{2}}$ ∴ F = 5.05 newton.

12 ⓓ

13

∵ The sum of the moments of the forces about a point B = the moment of the resultant about the point B

∴ $F \times X = 8 \times (X + 1.5)$

, ∵ F = 8 + 30 = 38 newtons

∴ $38 \times X = 8\,(X + 1.5)$

∴ $38\,X - 8\,X = 12$

∴ $30\,X = 12$

∴ $X = \frac{12}{30} = 0.4$ cm.

14

The two forces (6 , 6) equivalent a couple its algebraic measure

$M_1 = 6 \times 70 \cos 30° = 210\sqrt{3}$ kg.wt.cm.

∴ The two forces (R , F) equivalent a couple its algebraic measure = M_2

∴ R = F

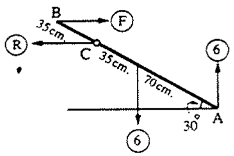
∴ $M_2 = -R \times 35 \sin 30°$

$= -17.5\,R$

, ∵ $M_1 + M_2 = 0$

∴ $210\sqrt{3} - 17.5\,R = 0$

∴ $R = 12\sqrt{3}$ kg.wt.



15

From the figure :

A = (0 , 0 , 15) , B = (–5 , 5 , 0)

$\overline{AB} = \overline{B} - \overline{A} = (-5 , 5 , -15)$

$\overline{F} = F \times \dfrac{\overline{AB}}{\|\overline{AB}\|} = 15\sqrt{11} \times \dfrac{(-5 , 5 , -15)}{\sqrt{(-5)^2 + (5)^2 + (-15)^2}}$

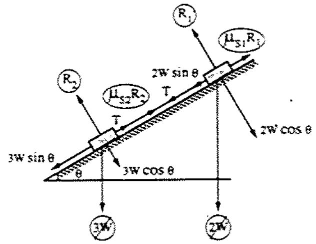
$= (-15 , 15 , -45)$

$\overline{M}_O = \overline{OA} \times \overline{F} = (0 , 0 , 15) \times (-15 , 15 , -45)$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 15 \\ -15 & 15 & -45 \end{vmatrix} = -225\,\hat{i} - 225\,\hat{j}$

16 ⓓ

17



The body whose coefficient of friction is the smaller should be put below the body whose coefficient of friction is the greater , so that the two bodies moves together and the string is taut

- With respect to the body whose weight is 3 w

∵ The body is about to move downwards

∴ $T + \mu_{S2}\,R_2 = 3\,w \sin\theta$, $R_2 = 3\,w \cos\theta$

∴ $T = 3\,w \sin\theta - \frac{1}{2}\,w \cos\theta$ (1)

- With respect to the body whose weight = 2 w

∵ The body is about to move downwards

∴ $\mu_{S1}\,R_1 = T + w \sin\theta$, $R_1 = 2\,w \cos\theta$

∴ $T = \frac{1}{2}\,w \cos\theta - 2\,w \sin\theta$ (2)

From (1) and (2) :

∴ $3\,w \sin\theta - \frac{1}{2}\,w \cos\theta = \frac{1}{2}\,w \cos\theta - 2\,w \sin\theta$

∴ $5\,w \sin\theta = w \cos\theta$ ∴ $\dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{5}$

∴ $\tan\theta = \frac{1}{5}$

18

[a] Let BC = X

∴ AC = 90 – X

∴ $(90 - X)^2 = (30)^2 + X^2$

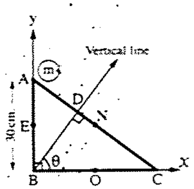
∴ $8100 + X^2 - 180\,X$

$= 900 + X^2$

∴ $180\,X = 7200$

∴ $X = 40$ cm.

∴ BC = 40 cm. , AC = 50 cm.



We can represent the mass of the wire by 3 masses by ratio 30 : 40 : 50

i.e. 3 : 4 : 5 at the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$

∴ The mass at E = 150

The mass at 0 = 200 gm. , mass at N = 250 gm.

Mass	E	O	N	A
	150	200	250	m
x	0	20	20	0
y	15	0	15	30

Then coordinates of centre of gravity of the system is

$X_G = \dfrac{200 \times 20 + 250 \times 20}{150 + 200 + 250 + m} = \dfrac{9000}{600 + m}$

$y_G = \dfrac{150 \times 15 + 250 \times 15 + m \times 30}{150 + 200 + 250 + m}$

$= \dfrac{6000 + 30\,m}{600 + m}$

From the figure :

$BD = \dfrac{30 \times 40}{50} = 24$ cm. ∴ CD = 32 cm.

, $\tan\theta = \dfrac{CD}{DB} = \dfrac{32}{24} = \dfrac{4}{3}$

∴ $\dfrac{y_G}{X_G} = \dfrac{6000 + 30\,m}{9000} = \dfrac{600 + 3\,m}{900} = \dfrac{4}{3}$

∴ 3600 = 1800 + 9 m ∴ m = 200 gm.

[b] $X_G = \dfrac{m_1 X_1 + m_2 X_2}{m_1 + m_2} = \dfrac{6 \times 0 + 12 \times 90}{6 + 12} = 60$

, $y_G = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \dfrac{6 \times 0 + 12 \times 0}{6 + 12} = 0$

∴ Centre of gravity = (60 , 0)

i.e. centre of gravity lies at distance 60 cm. from the body (6 kg.)

Model 10

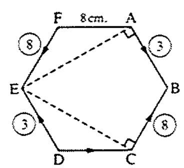
1 ⓑ **2** ⓑ **3** ⓓ **4** ⓓ

5

[a] From the shape property

$AE = CE = 8\sqrt{3}$ cm.

∴ The two forces (3 , 3) equivalent a couple its algebraic measure M_1



∴ $M_1 = -3 \times 8\sqrt{3} = -24\sqrt{3}$

∴ The two forces (8 , 8) equivalent a couple its algebraic measure = M_2

∴ $M_2 = 8 \times 8\sqrt{3} = 64\sqrt{3}$

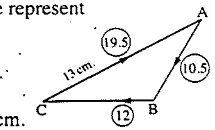
∴ The system equivalent a couple algebraic measure of its moment = $-24\sqrt{3} + 64\sqrt{3}$

$= 40\sqrt{3}$ kg.wt.cm.

[b] ∵ The sides of the triangle represent the forces completeiy.

∴ $\dfrac{10.5}{AB} = \dfrac{12}{BC} = \dfrac{19.5}{13} = \dfrac{3}{2}$

∴ AB = 7 cm. , BC = 8 cm.



∴ $\cos B = \dfrac{c^2 + a^2 - b^2}{2\,c \times a} = \dfrac{49 + 64 - 169}{2 \times 7 \times 8} = -\dfrac{1}{2}$

∴ m (∠ B) = 120°

∴ The sides are in the same way round

∴ The system of forces is equivalent to a couple whose norm of moment

$= 2 \times$ the area of Δ ABC $\times \frac{3}{2}$

$= 2 \times \frac{1}{2} \times 7 \times 8 \times \sin 120° \times \frac{3}{2}$

$= 42\sqrt{3}$ newton. cm.

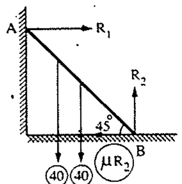
6 ⓐ **7** ⓐ

8

The equation of equilibrium :

$R_1 = \mu R_2$, $R_2 = 80$

, M_B = zero



∴ $40 \times \frac{\ell}{2} \cos 45° + 40$

$\times \frac{3}{4}\,\ell \cos 45° - R_1\,\ell \sin 45° = 0$

∴ $20 + 30 = R_1$ ∴ $R_1 = 50$

∴ $50 = \mu \times 80$ ∴ $\mu = \frac{5}{8}$

- If the boy wants to complete the ascending to the top

, the equations of equilibrium :

$R_1 = F + \frac{5}{8}\,R_2$, $R_2 = 80$

M_B = zero

$\therefore 40 \times \frac{\ell}{2} \cos 45° + 40 \times \ell \cos 45°$

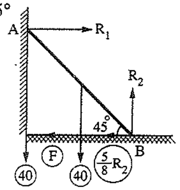
$- R_1 \times \ell \sin 45° = \text{zero}$

$\therefore 20 + 40 = R_1$

$\therefore R_1 = 60$

$\therefore 60 = F + \frac{5}{8} \times 80$

$\therefore F = 60 - 50 \qquad \therefore F = 10$ kg.wt.



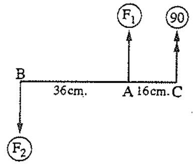
9 ⓑ **10** ⓑ

11

$\because$ Sum of moments of the forces about B = moment of the resultant about B

$\therefore F_1 \times 36 = 90 \times 52$

$\therefore F_1 = 130$ kg.wt.



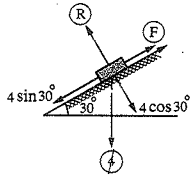
12 ⓐ

13

Equations of equilibrium :

$R = 4 \cos 30° = 2\sqrt{3}$ kg.wt.

$\therefore$ Component of the weight in direction line of greatest slope = 4 sin 30° = 2 kg.wt.



$\therefore F_s = \mu_s R = \frac{\sqrt{3}}{2} \times 2\sqrt{3} = 3$ kg.wt.

$\because F_s >$ the component of the weight in direction of line of greatest slope

$\therefore$ The body is in rest on the plane

$\therefore$ Smallest force which make the body is about to move downward = 3 − 2 = 1 kg.wt.

Greatest force which makes the body about to move upwards = 3 + 2 = 5 kg.wt.

14

The two forces equivalent a couple (5 , 5) algebraic measure of its moment = M_1

$M_1 = 5 \times 12 \times \cos 30° = 30\sqrt{3}$ kg.wt.cm.

$\therefore$ (F , R) make a couple algebraic measure of its moment = M_2

$\therefore F = R$

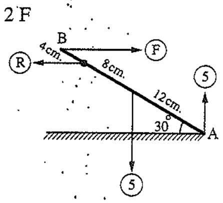
, $M_2 = - F \times 4 \times \sin 30° = - 2F$

$\therefore$ The rod is in equilibrium

$\therefore M_1 + M_2 = 0$

$\therefore 30\sqrt{3} - 2F = 0$

, $F = 15\sqrt{3}$ kg.wt.



15

Smallest force when θ = 90°

$\therefore M_O = 400 \qquad \therefore F \times 20 = 400$

$\therefore$ F = 20 N and it is the smallest force makes a rotation.

16 ⓑ

17

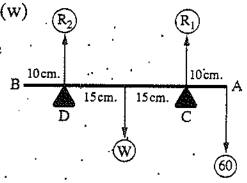
Let the weight of the rod (w)

$\because$ The rod about to rotate about the point (C)

$\therefore R_2 = 0$

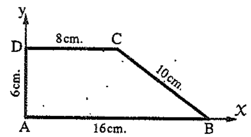
$\therefore M_C = 0$

$\therefore 60 \times 10 = w \times 15 \qquad \therefore$ w = 40 newton.



18

[a] Let $\overrightarrow{AB}$, $\overrightarrow{AD}$ the perpendicular directions



$\therefore$ BC = 40 − (8 + 6 + 16) = 10 cm.

The ratio between the lengths are 16 : 6 : 8 : 10

i.e. equals 8 : 3 : 4 : 5

Mass	8 m	3 m	4 m	5 m
x	8	0	4	12
y	0	3	6	3

$x_G = \frac{8\,m \times 8 + 4\,m \times 4 + 5\,m \times 12}{20\,m} = 7$

, $y_G = \frac{3\,m \times 3 + 4\,m \times 6 + 5\,m \times 3}{20\,m} = 2.4$

Distant of centre of gravity from $\overleftrightarrow{AB}$ = 2.4 cm.

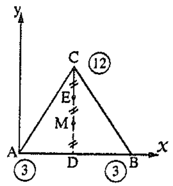
and its distant from $\overleftrightarrow{AD}$ = 7 cm.

[b] Distributing the mass 3 kg. among the vertices A , B , C to make the mass at

A = 3 kg.

, at B = 3 kg.

, at C = 12 kg.



, consider $\overrightarrow{AX}$, $\overrightarrow{Ay}$ two orthogonal directions

, let the side length of the triangle be ℓ

$\therefore CD = \frac{\sqrt{3}}{2}\ell$

$\therefore M = \left(\frac{1}{2}\ell , \frac{\sqrt{3}}{6}\ell\right)$, $E = \left(\frac{1}{2}\ell , \frac{\sqrt{3}}{3}\ell\right)$

	A	B	C
m	3	3	12
x	0	ℓ	$\frac{1}{2}\ell$
y	0	0	$\frac{\sqrt{3}}{2}\ell$

$\therefore x_G = \frac{3 \times \ell + 12 \times \frac{1}{2}\ell}{3 + 3 + 12} = \frac{1}{2}\ell$

, $y_G = \frac{12 \times \frac{\sqrt{3}}{2}\ell}{3 + 3 + 12} = \frac{\sqrt{3}}{3}\ell$

$\therefore$ Centre of gravity of the system = $\left(\frac{1}{2}\ell , \frac{\sqrt{3}}{2}\ell\right)$

, $\because$ E (midpoint of $\overline{CM}$) = $\left(\frac{1}{2}\ell , \frac{\sqrt{3}}{2}\ell\right)$

$\therefore$ The centre of gravity is midpoint of $\overline{MC}$

Model 11

1 ⓒ **2** ⓑ **3** ⓐ **4** ⓐ

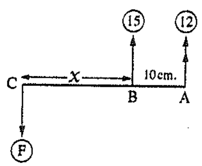
5

[a] Let the distant between two forces are X , sum of moments of the forces about C = moment of the resultant about C

$\therefore 12 \times (10 + X) = 15 \times X$

$120 + 12X = 15X$

$\therefore X = 40$ cm.



[b] $\because \overline{CA} = \overline{A} - \overline{C} = (0 , 2) - (2 , 3) = (-2 , -1)$

$\therefore \overline{M}_C = (-2 \times -4 + 1 \times 3)\hat{k}\,(8 + 3)\hat{k} = 11\hat{k}$

$\because \overline{DA} = \overline{A} - \overline{D} = (0 , 2) - (-2 , 1) = (2 , 1)$

$\therefore \overline{M}_D = (2 \times -4 - 1 \times 3)\hat{k} = -11\hat{k}$

$\because \overline{M}_C = - \overline{M}_D$

$\therefore$ Line of action of $\overline{F}$ bisects $\overline{DC}$

6 ⓒ

7

[a] Moment of the force $\overline{F}$ about to the origin point

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 3 & -4 & -12 \end{vmatrix} = -20\hat{i} - 9\hat{j} - 2\hat{k}$

[b] $\because \overline{BA} = (-1 , 2 , 1) - (3 , 4 , 0) = (-4 , -2 , 1)$

$\therefore$ Moment of the force $\overline{F}$ respect to (B)

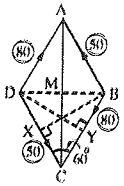
$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & -2 & 1 \\ 3 & -4 & -12 \end{vmatrix} = 28\hat{i} - 45\hat{j} + 22\hat{k}$

8 ⓓ **9** ⓑ

10

BX = DY = 12 sin 60°

$= 6\sqrt{3}$ cm.



$\because$ The two forces (50 , 50) form a couple , its algebraic measure M_1 where :

$M_1 = 50 \times 6\sqrt{3} = 300\sqrt{3}$ gm.wt. cm.

$\because$ The two forces (80 , 80) form a couple , its algebraic measure is M_2 where :

$M_2 = - 80 \times 6\sqrt{3} = - 480\sqrt{3}$ gm.wt.cm.

$M = 300\sqrt{3} - 480\sqrt{3} = - 180\sqrt{3}$ gm.wt.cm.

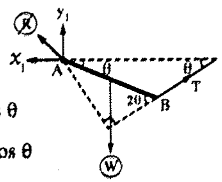
$\therefore \|\overline{M}\| = 180\sqrt{3}$ gm.wt.cm.

11 ⓒ **12** ⓐ

13

Let the components of the reaction of the hinge be X_1 and y_1

The equations of equilibrium :



$X_1 = T\cos\theta$ (1)

$y_1 = w - T\sin\theta$ (2)

, M_A = zero

$\therefore T \times \ell \sin 2\theta = w \times \frac{\ell}{2}\cos\theta$

$\therefore T \times 2\sin\theta\cos\theta = \frac{1}{2} w\cos\theta$

i.e. $T = \frac{1}{4} w \csc\theta$

Substituting in (1) : $\therefore X_1 = \frac{1}{4} w\cot\theta$

Substituting in (2) : $\therefore y_1 = w - \frac{1}{4} w = \frac{3}{4} w$

$\therefore$ R (the reaction of the hinge) $= \sqrt{X_1^2 + y_1^2}$

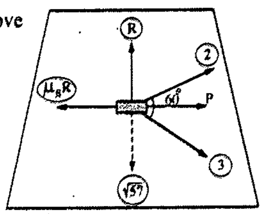
$\therefore R = \sqrt{\frac{1}{16} w^2 \cot^2\theta + \frac{9}{16} w^2} + = \frac{1}{4} w\sqrt{\cot^2\theta + 9}$

$= \frac{1}{4} w\sqrt{(\csc^2\theta - 1) + 9} = \frac{1}{4} w\sqrt{8 + \csc^2\theta}$

14

$P = \sqrt{4 + 9 + 2 \times 2 \times 3 \times \frac{1}{2}}$

$= \sqrt{19}$



$\because$ The body about to move

$\therefore \sqrt{19} = \mu_s R$

$\because R = \sqrt{57}$

$\therefore \sqrt{19} = \sqrt{57}\,\mu_s$

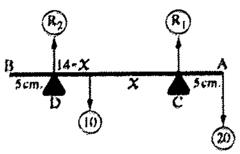
$\therefore \mu_s = \frac{1}{\sqrt{3}}$

$\therefore \tan\lambda = \frac{1}{\sqrt{3}} \Rightarrow$ measure of angle of friction = 30°

15

Let the centre of gravity of the rod be at distant X cm.

from the point (C)



$\therefore R_2 = 0$

$M_C = 0 : 20 \times 5 = 10 \times X$

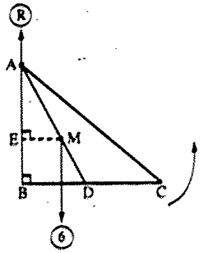
$\therefore X = 10$ cm.

Distant of the centre of gravity of rod from A = 15 cm.

16 ⓓ

17

$\because \overline{AD}$ is median



$\therefore$ BD = 7.5 cm.

from the similarity of the two triangles AEM , ABD

$\frac{ME}{BD} = \frac{AM}{AD} = \frac{2}{3}$

$\therefore ME = \frac{2}{3} \times 7.5 = 5$ cm.

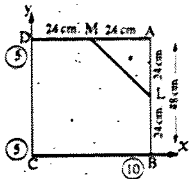
The two forces (6 , R) equivalent a couple the algebraic measure of its moment M_1

$M_1 = -6 \times 5 = -30$ newton.cm.

$\because$ The lamina is in equilibrium

$\therefore M_1 + M_2 = 0$ $\therefore M_2 = 30$ newton.cm.

18



Area of Δ AML :

Area of square ABCD

$= \frac{1}{2} \times 24 \times 24 : 48 \times 48$

$= 1 : 8$

$\therefore$ Mass of Δ AML $= \frac{1}{8} \times 40 = 5$ gm.

and its centre of gravity

$= \left(\frac{24 + 48 + 48}{3}, \frac{24 + 48 + 48}{3}\right) = (40, 40)$

$\therefore$ The mass at B = 10 gm. , at D = 5 gm.

, at C = 5 gm.

	B	C	D	square	triangle
Mass	10	5	5	40	−5
X	48	0	0	24	40
y	0	0	48	24	40

$\therefore X_G = \frac{10 \times 48 + 40 \times 24 - 5 \times 40}{10 + 5 + 5 + 40 - 5} = \frac{248}{11}$

$, y_G = \frac{5 \times 48 + 40 \times 24 - 5 \times 40}{10 + 5 + 5 + 40 - 5} = \frac{200}{11}$

$\therefore \tan\theta = \frac{200}{248} = \frac{25}{31}$

Model 12

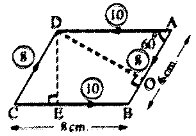
1 ⓒ **2** ⓒ

3

$\because DE = 6\sin 60° = 3\sqrt{3}$ cm.

$, DO = 8\sin 60° = 4\sqrt{3}$ cm.

$\because$ The two forces (10 , 10) form a couple , its algebraic measure is M_1 where :



$M_1 = 10 \times 3\sqrt{3} = 30\sqrt{3}$ newton.cm.

$\because$ The two forces (8 , 8) form a couple , its algebraic measure is M_2 where :

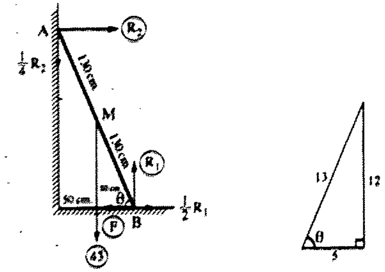
$M_2 = -8 \times 4\sqrt{3} = -32\sqrt{3}$ newton.cm.

$\therefore M = M_1 + M_2 = -2\sqrt{3}$ newton.cm.

$\therefore \|\overline{M}\| = 2\sqrt{3}$ newton.cm.

4 ⓑ **5** ⓓ **6** ⓐ **7** ⓓ **8** ⓓ

9



The equations of equilibrium :

$R_1 = \frac{1}{4} R_2 + 43$ (1)

$R_2 + \frac{1}{2} R_1 = F$ (2)

$M_B = 0$

$43 \times 50 + \frac{1}{4} R_2 \times 100 - R_2 \times 260 \sin\theta = 0$

$\therefore 215 R_2 = 2150$

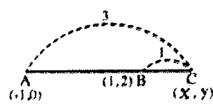
$\therefore R_2 = 10$ newton.

From (1) : $R_1 = \frac{1}{4} \times 10 + 43 = 45.5$ newton.

$, F = 10 + \frac{1}{2} \times 45.5 = 32.75$ newton.

10 ⓑ

11



$\overline{R} = \overline{F_1} + \overline{F_2} = -6\hat{i} + 2\hat{j}$

$\because \overline{F_2} = -9\hat{i} + 3\hat{j}$

$= -3(3\hat{i} - \hat{j}) = -3\overline{F_1}$

$\therefore$ The two forces $\overline{F_1}$, $\overline{F_2}$ are parallel and in opposite direction

Let the resultant acts in the point (C) = (X, y)

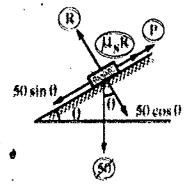
$\therefore$ C divides $\overline{AB}$ externally where $\frac{AC}{BC} = \frac{3}{1}$

$\therefore X = \frac{3 \times 1 - 1 \times -1}{3 - 1} = 2$

$y = \frac{3 \times 2 - 1 \times 0}{3 - 1} = 3$ C = (2 , 3)

12

[a] When the body is about to move downwards (the least force)



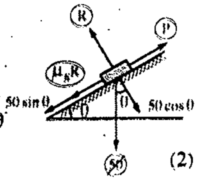
$\therefore P + \mu_s R = 50\sin\theta$

$, R = 50\cos\theta$

$\therefore 10 + \mu_s \times 50\cos\theta = 50\sin$

$\therefore \mu_s = \frac{50\sin\theta - 10}{50\cos\theta}$ (1)

and when the body is about to move upwards (the greatest force)



$\therefore P = \mu_s R + 50\sin\theta$

$, R = 50\cos\theta$

$40 = \mu_s \times 50\cos\theta + 50\sin\theta$

$\therefore \mu_s = \frac{40 - 50\sin\theta}{50\cos\theta}$ (2)

From (1) , (2) : $\frac{50\sin\theta - 10}{50\cos\theta} = \frac{40 - 50\sin\theta}{50\cos\theta}$

$\therefore 100\sin\theta = 50$ $\therefore \sin\theta = \frac{1}{2}$

$\therefore \theta = 30°$, $\mu_s = \frac{\sqrt{3}}{5}$

[b] $\overline{r} = \overline{OA} = (-1, 3)$

$\overline{M}_O = (-1 \times 4 - 3 \times 3)\hat{k} = (-4 - 9)\hat{k} = -13\hat{k}$

$\ell = \frac{\|\overline{M}_O\|}{F} = \frac{13}{\sqrt{3^2 + 4^2}} = \frac{13}{5} = 2.6$ length unit.

13 ⓒ

14

From the figure :

A = (0 , 2 , 2) , B = (−1 , 2 , 0)

$\overline{AB} = \overline{B} - \overline{A} = (-1, 0, -2)$

$\overline{F} = F \times \frac{\overline{AB}}{\|\overline{AB}\|} = 14\sqrt{5} \times \frac{(-1, 0, -2)}{\sqrt{(-1)^2 + (-2)^2}}$

$= (-14, 0, -28)$

$\overline{M}_o = \overline{OA} \times \overline{F} = (0, 2, 2) \times (-14, 0, -28)$

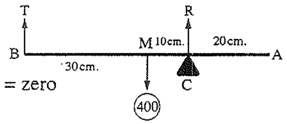
$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 2 \\ -14 & 0 & -28 \end{vmatrix} = -56\hat{i} - 28\hat{j} + 28\hat{k}$

15 ⓐ

16

From equilibrium properties :

$R + T = 400$ (1)



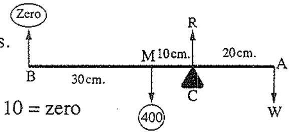
$M_C = 0$

$\therefore 400 \times 10 - T \times 40 = \text{zero}$

$\therefore T = 100$ gm.wt.

From (1) : $\therefore R = 300$ gm.wt.

When the tension at B about to vanishes.



$M_C = 0$

$\therefore - w \times 20 + 400 \times 10 = \text{zero}$

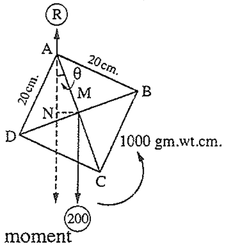
$w = 200$ gm.wt.

17

$AC = 20\sqrt{2}$

$AM = 10\sqrt{2}$

$\because$ Lamina is in equilibrium under action of two couples



$\therefore (200 , R)$ is a couple , algebraic measure of its moment

$= - 1000 \text{ gm.wt.cm.} - 200 \times MA \sin \theta = - 1000$

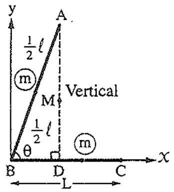
$- 200 \times 10\sqrt{2} \sin \theta = - 1000$

$\therefore \sin \theta = \frac{1}{2\sqrt{2}}$ $\qquad \therefore \theta = 20° 42' 17''$

18

[a]

Mass	m	m
x	$\frac{\ell}{2}$	$\frac{1}{2} \ell \cos \theta$
y	0	$\frac{1}{2} \ell \sin \theta$



$$x_G = \frac{\frac{\ell}{2} \times m + \frac{1}{2} \ell m \cos \theta}{m + m} = \frac{1}{4}\ell + \frac{1}{4}\ell \cos \theta$$

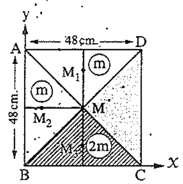
$$y_G = \frac{m \times \frac{1}{2} \ell \sin \theta}{m + m} = \frac{1}{4}\ell \sin \theta$$

$\because \overline{BC}$ is horizontal

$$\therefore \cos \theta = \frac{BD}{AB} = \frac{\frac{1}{4}\ell + \frac{1}{4}\ell \cos \theta}{\ell} = \frac{1}{4} + \frac{1}{4} \cos \theta$$

$\therefore \cos \theta = \frac{1}{3}$

[b] Let $\overrightarrow{BC}$, $\overrightarrow{BN}$ are x , y-axes.



Let mass of Δ MDA = m acting at M_1

, mass of Δ MAB = m acting at M_2 , mass of Δ MBC = 2 m acting at M_3 , from the figure :

$MM_1 = MM_2 = MM_3 = \frac{2}{3} \times 24 = 16$ cm.

Mass	M_1	M_2	M_3
	m	m	2 m
x	24	8	24
y	40	24	8

$$x_G = \frac{24 m + 8 m + 24 \times 2 m}{m + m + 2 m} = 20$$

$$y_G = \frac{40 m + 24 m + 2 m \times 8}{m + m + 2 m} = 20$$

$\therefore$ Centre of gravity = (20 , 20)

i.e. distant 20 cm. from each of $\overline{AB}$, $\overline{BC}$

Model 13

1 ⓑ **2** ⓑ **3** ⓓ **4** ⓐ

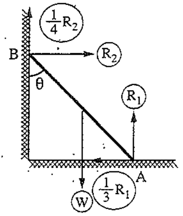
5

Let the rod length be 2 L and inclined to the vertical at an angle θ

From the equilibrium equations :

$\frac{1}{3} R_1 = R_2$ (1)

$R_1 + \frac{1}{4} R_2 = w$ (2)



By substituting from (1) in (2) :

$\therefore R_1 + \frac{1}{4}\left(\frac{1}{3} R_1\right) = w$

$\therefore \frac{13}{12} R_1 = w$

$\therefore R_1 = \frac{12}{13} w$ (3)

By substituting in (1) : $\therefore R_2 = \frac{4}{13} w$ (4)

$M_A = 0$:

$L \times w \times \sin \theta - 2 L \times R_2 \cos \theta - 2 L \times \frac{1}{4} R_2 \sin \theta = 0$

$w \sin \theta - 2 R_2 \cos \theta - \frac{1}{2} R_2 \sin \theta = 0$ (5)

by substituting from (4) in (5) :

$w \sin \theta - 2\left(\frac{4}{13} w\right) \cos \theta - \frac{1}{2}\left(\frac{4}{13} w\right) \sin \theta = 0$

$\sin \theta - \frac{8}{13} \cos \theta - \frac{2}{13} \sin \theta = 0$

$\frac{11}{13} \sin \theta = \frac{8}{13} \cos \theta$

$\tan \theta = \frac{8}{11}$

6 ⓑ **7** ⓓ

8

[a] Taking the moments about the origin

$$M = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -2 \\ 2 & b & 1 \end{vmatrix}$$

$= (3 + 2 b) \hat{i} - 3 \hat{j} + (- b - 6) \hat{k}$

$\therefore 3 + 2 b = - 3$ $\qquad \therefore b = - 3$

, the perpendicular length drawn from origin to the line of action of the force

$= \frac{\sqrt{9 + 9 + 9}}{\sqrt{1 + 9 + 4}} = \frac{3\sqrt{42}}{14}$

[b] With respect to C :

$\vec{r} = \overrightarrow{CA} = \vec{A} - \vec{C} = (0 , 2) - (2 , 3) = (- 2 , - 1)$

$\therefore \overrightarrow{M_C} = (- 2 \times - 4 + 1 \times 3) \hat{k}$

$\therefore \overrightarrow{M_C} = (8 + 3) \hat{k} = 11 \hat{k}$

with respect to E :

$\vec{r} = \overrightarrow{EA} = \vec{A} - \vec{E} = (0 , 2) - (5 , - 1) = (- 5 , 3)$

$\therefore \overrightarrow{M_E} = (- 5 \times - 4 - 3 \times 3) \hat{k}$

$\therefore \overrightarrow{M_E} = (20 - 9) \hat{k} = 11 \hat{k}$ $\qquad \because \overrightarrow{M_C} = \overrightarrow{M_E}$

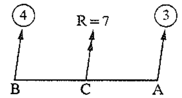
$\therefore$ The line of action of force $\vec{F} \parallel \overline{CE}$

9 ⓐ **10** ⓑ

11

Before the movement of the force

$3 \times AC = 4 \times BC$ (1)



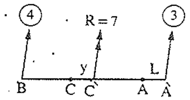
After the force 3 had moved distance ℓ in direction $\overrightarrow{BA}$

Let the resultant had moved y in the same direction

$\therefore 3 \times A\grave{}\, C\grave{} = 4 \times BC\grave{}$

$\therefore 3 \times (AC + \ell - y)$

$= 4 \times (BC + y)$



$\therefore 3 AC + 3 \ell - 3 y = 4 BC + 4 y$

By substituting from (1) : $3 \ell = 7 y$ $\qquad y = \frac{3}{7} \ell$

12

[a] $\because \vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} = (6 + (- 1) + (- 5)) \hat{i} + (3 + (- 4) + 1) \hat{j} = \vec{0}$

$\therefore$ The system is equivalent to a couple or in equilibrium

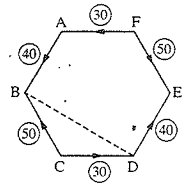
$\because \overrightarrow{M_A} = \overrightarrow{AB} \times \vec{F_2} + \overrightarrow{AC} \times \vec{F_3}$

$= (1 , 8) \times (- 1 , - 4) + (3 , 2) \times (- 5 , 1)$

$= (- 4 + 8 + 3 + 10) \hat{k} = 17 \hat{k}$

$\therefore$ The system represents a couple , the magnitude of its moment = 17 moment unit.

[b] $\because$ The two forces (40 , 40) form a couple whose algebraic measure (M_1)



$\therefore M_1 = 40 \times 15\sqrt{3}$

$= 600\sqrt{3}$ newton.cm.

$\because$ The two forces (50 , 50) form a couple whose algebraic measure (M_2)

$\therefore M_2 = - 50 \times 15\sqrt{3} = - 750\sqrt{3}$ newton.cm.

$\because$ The two forces (30 , 30) form a couple whose algebraic measure (M_3)

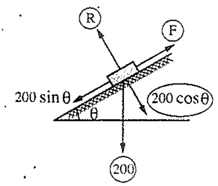
$\therefore M_3 = 30 \times 15\sqrt{3} = 450\sqrt{3}$ newton.cm.

$\therefore M = M_1 + M_2 + M_3 = 300\sqrt{3}$ newton.cm.

13 ⓑ **14** ⓑ

15

From the equilibrium equations :



$R = 200 \cos \theta = 200 \times \frac{\sqrt{3}}{2}$

$= 100\sqrt{3}$ gm.wt.

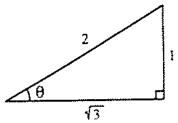
$F_s = \mu_s R = \frac{\sqrt{3}}{4} \times 100\sqrt{3} = 75$ gm.wt.

, $\because$ the component of weight

$= 200 \sin \theta = 200 \times \frac{1}{2} = 100$ gm.wt.

$\because$ The component of the weight

in direction of the line of greatest slope > F upward



∴ The friction force acts upwards

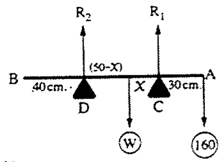
$F = 100 - 25 = 75 = F_s$

∴ The body is about to move downwards.

16

In the first case :

∵ The rod is about to overturn about C



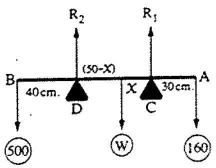
∴ $R_2 = 0$

$M_C = 0 : 160 \times 30 = X \times w$

$4800 = X\, w$ (1)

In the second case :

∵ The rod is about to overturn about D



∴ $R_1 = 0$

$M_D = 0 : 160 \times 80 + w\,(50 - X)$

$= 500 \times 40$

$12800 + 50\,w - w\,X = 20000$ (2)

By substituting from (1) : $50\,w - 4800 = 7200$

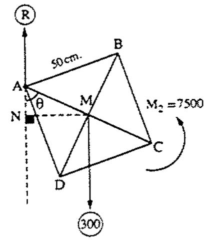
∴ w = 240 gm.wt.

17

$AC = \sqrt{50^2 + 50^2} = 50\sqrt{2}$ cm.

∴ $AM = 25\sqrt{2}$ cm.

∵ The lamina is in equilibrium under effect of a couple whose algebraic measure 7500 gm.wt.cm.



∴ (R , 300) form a couple whose algebraic measure = 7500 gm.wt.cm.

∴ $M_1 + M_2 = 0$ $M_1 = - M_2$

∴ $300 \times 25\sqrt{2} \sin\theta = 7500$

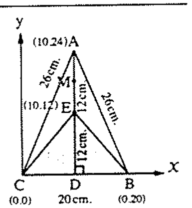
$\sin\theta = \frac{1}{\sqrt{2}}$ ∴ θ = 45° or 135°

18

Let the two perpendicular directions be $\overrightarrow{CX}$, $\overrightarrow{Xy}$, C is the origion

From the figure :

$AD = \sqrt{26^2 - 10^2} = 24$ cm.



The area of Δ ABC = $\frac{1}{2} \times 20 \times 24 = 24\ cm^2$

and its centre = $\left(\frac{20 + 0 + 10}{3}, \frac{0 + 0 + 24}{3}\right) = (10 , 8)$

and the area of Δ EBC = $\frac{1}{2} \times 20 \times 12 = 120$

and its centre = $\left(\frac{0 + 20 + 10}{3}, \frac{0 + 0 + 12}{3}\right) = (10 , 4)$

$X_G = \frac{240 \times 10 - 120 \times 10}{240 - 120} = 10$

$y_G = \frac{240 \times 8 - 120 \times 4}{240 - 120} = 12$

m	240	− 120
X	10	10
y	8	4

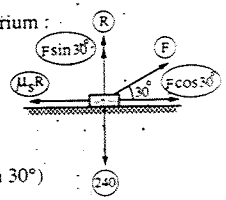
∴ E is the centre of gravity of the remaining part.

Model 14

1

From the equation of equilibrium :

F sin 30° + R = 240 kg.wt.



∴ R = 240 − F sin 30°

∴ F cos 30° = μ_s R

∴ $\frac{\sqrt{3}}{2} F = 0.3 \times (240 - F \sin 30°)$

$\frac{\sqrt{3}}{2} F = 72 - 0.15\,F$ ∴ F = 70.86 kg.wt.

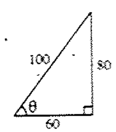
2 ⓑ **3** ⓐ **4** ⓓ

5 ⓐ **6** ⓑ

7

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$



∵ The rod is in equilibrium

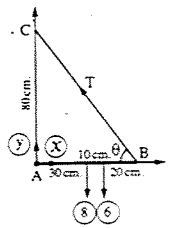
∴ $M_A = 0$

∴ $-8 \times 30 - 6 \times 40 + 60 \times T \times \sin\theta = 0$

∴ $60 \times T \times \frac{4}{5} = 480$

∴ T = 10 newton.

From equation of equilibrium :



$y + T \sin\theta = 6 + 8$

∴ $y + 10 \times \frac{4}{5} = 14$

∴ y = 6

, $X = T \cos\theta$

∴ $X = 10 \times \frac{3}{5} = 6$

∴ The reaction of the hinge $= \sqrt{X^2 + y^2} = \sqrt{36 + 36}$

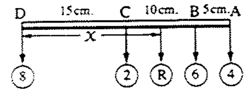
$= 6\sqrt{2}$ newton

8 ⓐ **9** ⓑ **10** ⓑ **11** ⓑ

12

[a] R = 4 + 6 + 2 + 8

= 20 kg.wt.



∵ The sum of moment of forces about D

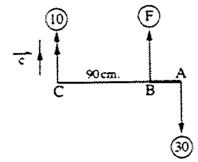
= moment of the resultant about D

∴ $4 \times 30 + 6 \times 25 + 2 \times 15 = 20 \times X$

∴ X = 15 cm.

∴ The resultant acts at a distance 15 cm. from the point D , 15 cm. from the point A

[b] F − 30 = 10



∴ F = 40 newton.

, 30 (AB + 90) = 40 × 90

∴ AB + 90 = 120

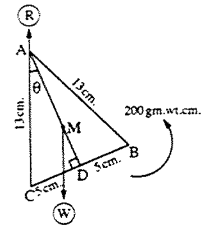
∴ AB = 30 cm.

13

From the figure :

$AD = \sqrt{169 - 25} = 12$ cm.

∴ $AM = \frac{2}{3} \times 12 = 8$ cm.



∵ The lamina is in equilibrium under the action of two couples

∴ The two forces (w , R) form a couple the algebraic measure of its moment = − 200 gm.wt.cm.

∴ $- w \times 8 \times \sin\theta = - 200$

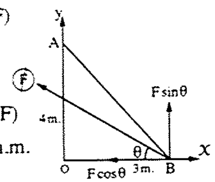
∴ $w \times 8 \times \frac{5}{13} = 200$

∴ w = 65 gm.wt.

14

∵ The moment of the force (F) with respect to B = 0

∴ The force passes through B



, ∵ the moment of the force (F) with respect to O = 84 newton.m.

∴ F sin θ × 3 = 84

∴ F sin θ = 28 (1)

, ∵ the moment of the force (F) with respect to A = − 100 newton.m.

∴ − F cos θ × 4 + F sin θ × 3 = − 100

By substituting from (1) :

∴ − 4 F cos θ + 28 × 3 = − 100

∴ F cos θ = 46 (2)

Divide (1) by (2) : tan θ = $\frac{28}{46} = \frac{14}{23}$

∴ θ = 31° 19 43

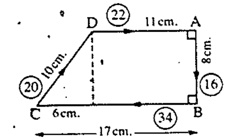
By substituting in (1) : F = 53.85 newton.

15

[a] ∵ The forces act in one cyclic order

, $\frac{22}{11} = \frac{16}{8} = \frac{34}{17}$

$= \frac{20}{10} = 2$



∴ The system is equivalent to a couple whose algebraic measure M_1

$M_1 = - 2 \times$ the area of the trapezium × 2

$= - 2 \times \frac{1}{2} (17 + 11) \times 8 \times 2$

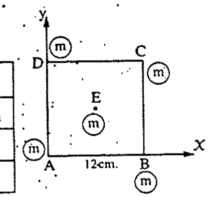
= − 448 gm.wt.cm,

∴ The magnitude of the couple moment

= 448 gm.wt.cm.

[b] Let the two perpendicular directions be $\overrightarrow{AB}$, $\overrightarrow{AD}$

Mass	A	B	C	D	E
	m	m	m	m	m
X	0	12	12	0	6
y	0	0	12	12	6



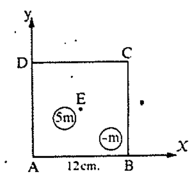
∴ $X_G = \frac{m \times 12 + m \times 12 + m \times 6}{5\,m} = 6$

, $y_G = \frac{m \times 12 + m \times 12 + m \times 6}{5\,m} = 6$

∴ The centre of gravity = (6 , 6)

Determine the centre of gravity after the mass at B had removed

Mass	E	B
	5 m.	− m.
X	6	12
y	6	0



$\therefore \bar{x}_G = \frac{5m \times 6 - m \times 12}{5m - m} = 4.5$

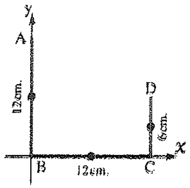
$, \bar{y}_G = \frac{5m \times 6}{5m - m} = 7.5$

$\therefore \vec{G} = (4.5 , 7.5)$

16 ⓓ

17

Let the two perpendicular directions be in direction of $\overrightarrow{BC}$, $\overrightarrow{BA}$



Mass	12	6	12
x	6	12	0
y	0	3	6

$x_G = \frac{12 \times 6 + 6 \times 12 + 12 \times 0}{12 + 6 + 12} = 4.8$

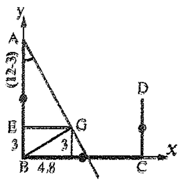
$, y_G = \frac{12 \times 0 + 6 \times 3 + 12 \times 6}{12 + 6 + 12} = 3$

$\therefore$ The centre of gravity of the wire at 4.8 cm. for from $\overline{AB}$ and 3 cm. for from $\overline{BC}$

In Case of suspending from A :

$\tan (\angle EAG) = \frac{4.8}{12 - 3}$

$\therefore m (\angle EAG) = 28° \, 4' \, 21''$



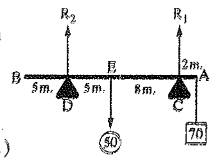
18

• When the man at A :

$\because 70 \times 2 < 50 \times 8$

$\therefore$ The board is in equilibrium

the equilibrium equations :



$R_1 + R_2 = 70 + 50$

$= 120$ kg.wt. (1)

$M_D = 0 \qquad \therefore - 70 \times 15 - 50 \times 5 + R_1 \times 13 = 0$

$\therefore R_1 = 100$ kg.wt.

From (1) : $R_2 = 20$ kg.wt.

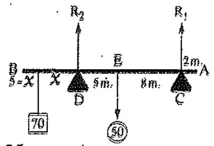
When the man moves (x) , the board is about to overturn about D

$\therefore R_1 = 0$

$\therefore M_D = 0$:

$\therefore 50 \times 5 = 70 \times x$

$\therefore x = \frac{25}{7}$ m.



$\therefore$ The man can move $= 15 + \frac{25}{7} = 18\frac{4}{7}$ m. from A without overturning the board.

Model 15

1 ⓒ **2** ⓒ **3** ⓑ

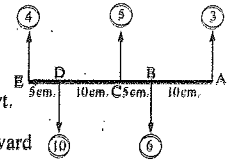
4 ⓓ **5** ⓓ **6** ⓑ

7

The sum of the forces upward

$= 3 + 5 + 4 = 12$ kg.wt.

, the sum of the forces downward $= 6 + 10 = 16$ kg.wt.



$\therefore$ The resultant acts downward

, $R = 16 - 12 = 4$ kg.wt.

Let the resultant act at a point x cm. for from E

$\because$ The sum of the moments about E = The resultant moment about E

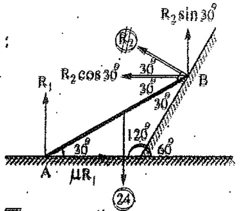
$\therefore 3 \times 30 - 6 \times 20 + 5 \times 15 - 10 \times 5 = - 4 \times x$

$\therefore x = 1.25$ cm.

$\therefore$ The resultant is at a distance 1.25 from E

8

The equilibrium equations :



$\mu R_1 = \frac{\sqrt{3}}{2} R_2$ (1)

$\frac{1}{2} R_2 + R_1 = 24$ (2)

$, M_A = \text{zero}$

$\therefore 24 \times \frac{\ell}{2} \times \frac{\sqrt{3}}{2} = R_2 \, \ell \times \frac{\sqrt{3}}{2}$

$\therefore R_2 = 12$

$\therefore$ The normal reaction of the inclined plane $= 12$ kg.wt.

From (2) : $\therefore 6 + R_1 = 24$, $\therefore R_1 = 18$

From (1) : $\therefore \mu \times 18 = \frac{\sqrt{3}}{2} \times 12$

$\therefore 18 \mu = 6\sqrt{3} \qquad \therefore \mu = \frac{\sqrt{3}}{3}$

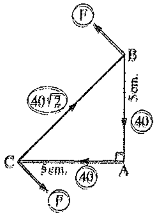
$\therefore$ The reaction of the ground ($\vec{R}$)

$= \sqrt{R_1^2 + (\mu R_1)^2} = R_1 \sqrt{1 + \mu^2} = 12\sqrt{3}$ kg.wt.

9

From the figure :

$BC = \sqrt{5^2 + 5^2} = 5\sqrt{2}$



$\because$ The forces act in one cyclic order

$, \because \frac{40}{5} = \frac{40}{5} = \frac{40\sqrt{2}}{5\sqrt{2}} = 8$

$\therefore$ The system represents a couple whose algebraic measure is M_1

$M_1 = - 2 \times$ the area of the triangle $\times 8$

$\therefore M_1 = - 2 \times \frac{1}{2} \times 5 \times 5 \times 8 = - 200$ newton.cm.

Let the two forces (F , F) act at B , C form a couple whose algebraic measure M_2

$, \because$ the system is in equilibrium $\qquad \therefore M_1 + M_2 = 0$

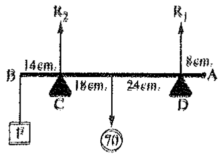
$\therefore M_2 = 200 \qquad \therefore 200 = F \times 5\sqrt{2}$

$\therefore F = 20\sqrt{2}$ newton.

$\therefore$ The two forces are $20\sqrt{2}$, $20\sqrt{2}$ newton.

10

[a] Let the weight should be suspended and makes the rod about to overturn is F



$\therefore R_1 = 0$

$\therefore 70 \times 18 = F \times 14$

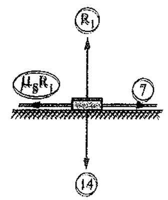
$\therefore F = 90$ newton.

[b] $\because$ The body is about to move

$\therefore \mu_s R_1 = 7$, $R_1 = 14$

$\therefore \mu_s \times 14 = 7$

$\therefore \mu_s = \frac{1}{2}$



The body after putting the weight

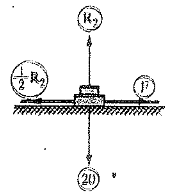
$\because$ The body is about to move

$\therefore \frac{1}{2} R_2 = F$

$, R_2 = 20$

$\therefore \frac{1}{2} \times 20 = F$

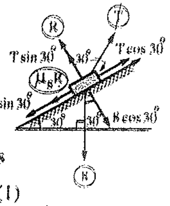
$\therefore F = 10$ kg.wt.



11 ⓑ

12

$\because$ The body is about to slide when the plane makes angle 30° to the horizontal



$\therefore \mu_s = \tan 30° = \frac{1}{\sqrt{3}}$

$, \because$ the body is about to move upwards

$\therefore$ The friction force act downwards

$\therefore T \cos 30° = \mu_s R + 8 \sin 30°$ (1)

$, R + T \sin 30° = 8 \cos 30°$

$\therefore R = 8 \cos 30° - T \sin 30°$ (2)

$\therefore \mu_s R = \frac{1}{\sqrt{3}} (8 \cos 30° - T \sin 30°)$

$= 4 - \frac{\sqrt{3}}{6} T$ (3)

By substituting in (1) :

$\therefore \frac{\sqrt{3}}{2} T = 4 - \frac{\sqrt{3}}{6} T + 4$

$\therefore \frac{2\sqrt{3}}{3} T = 8$

$\therefore T = 4\sqrt{3}$

From (2) :

$\therefore R = 8 \cos 30° - 4\sqrt{3} \times \sin 30° = 2\sqrt{3}$ kg.wt.

13

From the figure :

$A = (2 , 0 , 4)$, $D = (0 , 3 , 0)$

$\overrightarrow{AD} = \vec{D} - \vec{A} = (-2 , 3 , -4)$

$\vec{T} = T \times \frac{\overrightarrow{AD}}{\| \overrightarrow{AD} \|} = 10\sqrt{29} \times \frac{(-2 , 3 , -4)}{\sqrt{(-2)^2 + (3)^2 + (-4)^2}}$

$= (-20 , 30 , -40)$

$\therefore \overrightarrow{M_C} = \overrightarrow{CA} \times \overrightarrow{T}$

$\overrightarrow{CA} = \overrightarrow{A} - \overrightarrow{C} = (2, 0, 4)$

$\therefore \overrightarrow{M_C} = (2, 0, 4) \times (-20, 30, -40)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 4 \\ -20 & 30 & -40 \end{vmatrix} = -120\,\hat{i} + 60\,\hat{k}$$

14 ⓒ

15

[a] The area of the removed part Δ CMD :

The area of the square ABCD = 1 : 4

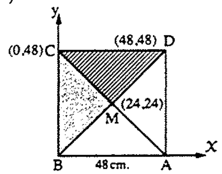
The centre of Δ CMD

$= \left(\frac{24+48+0}{3}, \frac{24+48+48}{3}\right) = (24, 40)$

The centre of Δ CMB

$= \left(\frac{0+24+0}{3}, \frac{0+24+48}{3}\right) = (8, 24)$

m	1	$-\frac{1}{4}$	$\frac{1}{4}$
x	24	24	8
y	24	40	24



$$x_G = \frac{1 \times 24 - \frac{1}{4} \times 24 + \frac{1}{4} \times 8}{1 - \frac{1}{4} + \frac{1}{4}} = 20 \text{ cm.}$$

$$y_G = \frac{1 \times 24 - \frac{1}{4} \times 40 + \frac{1}{4} \times 24}{1 - \frac{1}{4} + \frac{1}{4}} = 20 \text{ cm.}$$

$\therefore$ The centre of a gravity is 20 cm. far from $\overline{BA}$ and 20 cm. far from $\overline{BC}$

[b] $x_G = \frac{1 \times 2 + 2 \times -2 + 3 \times 0}{1+2+3} = \frac{-1}{3}$

$y_G = \frac{1 \times 3 + 2 \times 1 + 3 \times 1}{1+2+3} = \frac{4}{3}$

$\therefore$ The centre of gravity is $\left(-\frac{1}{3}, \frac{4}{3}\right)$

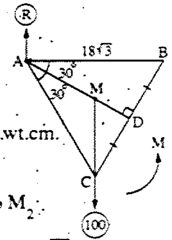
16 ⓒ **17** ⓒ

18

From the figure :

$AD = 18\sqrt{3} \cos 30° = 27$ cm.

$\therefore AM = 27 \times \frac{2}{3} = 18$ cm.



$\therefore$ The two forces (100 , R) form a couple whose algebraic measure is M_1

$M_1 = -100 \times 9\sqrt{3} = -900\sqrt{3}$ gm.wt.cm.

$\because$ The lamina is in equilibrium under effect of two couples M_1 , M_2 :

$\therefore M_1 + M_2 = 0$ $\therefore M_2 = 900\sqrt{3}$ gm.wt.cm.

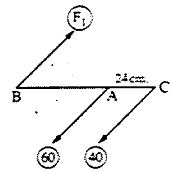
$\therefore$ The magnitude of couple moment

$M_2 = 900\sqrt{3}$ gm.wt.cm

Model 16

1 ⓒ **2** ⓒ

3



$R = F_2 - F_1$

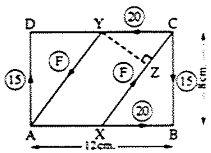
$\therefore 40 = 60 - F_1$

$\therefore F_1 = 20$ newton

$\therefore 60 \times 24 = 20\,(24 + AB)$

$24 + AB = 72$ $\therefore AB = 48$ cm.

4



$\because BX = 6$ cm. , $BC = 8$ cm.

$\therefore CX = \sqrt{(6)^2 + (8)^2} = 10$ cm.

$YZ = CY \sin(\angle YCX)$

$= 6 \times \frac{8}{10} = 4.8$ cm.

, $\because$ the two forces (20 , 20) form a couple , the algebraic measure of its moment is (M_1)

$\therefore M_1 = 20 \times 8 = 160$ newton.cm.

, the two forces (15 , 15) form a couple the algebraic measure of its moment is (M_2)

$\therefore M_2 = -15 \times 12 = -180$ newton.cm.

, the two forces (F , F) form a couple the algebraic measure of its moment is (M_3)

$\therefore M_3 = F \times 4.8$ newton.cm.

$\therefore 160 - 180 + 4.8\,F = 100$

$\therefore F = 25$ newton.

5 ⓑ

6

[a] $\overrightarrow{M_o} = \overrightarrow{r} \times \overrightarrow{F} = (-1, -8, n)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 1 \\ k & m & -2 \end{vmatrix}$$

$= (-\hat{i} - 8\hat{j} + n\hat{k})$

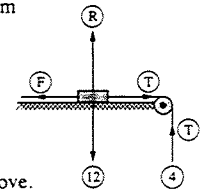
$= (-2 - 2m)\hat{i} + (k+6)\hat{j} + (3m - k)\hat{k}$

$= (-1, -8, n)$

$\therefore -2 - m = -1$ then $m = -1$

, $k + 6 = -8$ $k = -14$

[b] $\because$ The body is in equilibrium



$\therefore F = T = 4$ newton.

, $R = 12$

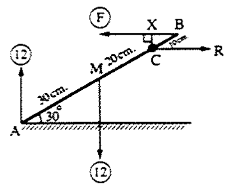
, $\because \mu_s R = \frac{1}{3} \times 12 = 4$

$\therefore F = \mu_s R = 4$

$\therefore$ The body is about to move.

7 ⓓ **8** ⓒ **9** ⓑ

10



$\because$ The two forces (12 , 12) form a couple the algebraic measure of its moment is (M_1)

, $M_1 = -12 \times 30 \cos 30°$

$= -180\sqrt{3}$ newton.cm.

, the couple is in equilibrium with only another couple

$\therefore$ The two forces (F , R) form a couple

$\therefore$ R acts horizontally in opposite direction of $\overrightarrow{F}$, R = F

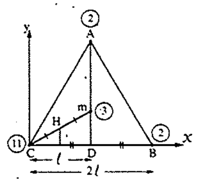
, $\because CX = 10 \sin 30° = 5$ cm.

$\therefore F = R = \frac{180\sqrt{3}}{5} = 36\sqrt{3}$ newton.

11

[a] Let the side length of the triangle = $2\,\ell$

Mass	3	2	2	11
x_G	ℓ	ℓ	2ℓ	0
y_G	$\frac{\sqrt{3}\,\ell}{3}$	$\sqrt{3}\,\ell$	0	0

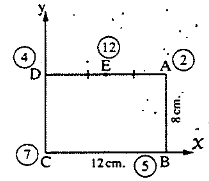


$$x_G = \frac{3 \times \ell + 2 \times \ell + 2 \times 2\ell + 11 \times 0}{3+2+2+11} = \frac{1}{2}\,\ell$$

$$y_G = \frac{3 \times \ell \times \frac{\sqrt{3}}{3} + 2 \times \sqrt{3}\,\ell + 2 \times 0 + 11 \times 0}{3+2+2+11} = \frac{\sqrt{3}}{6}\,\ell$$

$\therefore G\left(\frac{1}{2}\,\ell, \frac{\sqrt{3}}{6}\,\ell\right)$ which is the midpoint of $\overline{CM}$

[b]



Consider $\overrightarrow{CB}$, $\overrightarrow{CD}$ are two orthogonal directions

	A	E	D	C	B
m	2	12	4	7	5
x	12	6	0	0	12
y	8	8	8	0	0

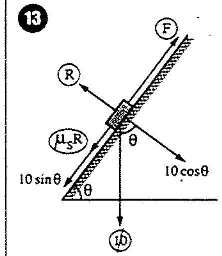
$x_G = \frac{2 \times 12 + 12 \times 6 + 5 \times 12}{2+12+4+7+5} = 5.2$

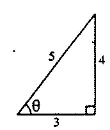
, $y_G = \frac{2 \times 8 + 12 \times 8 + 4 \times 8}{2+12+4+7+5} = 4.8$

$\therefore$ The centre of gravity = (5.2 , 4.8) with respect to the point C

12 ⓓ

13





When the body is about to move upwards

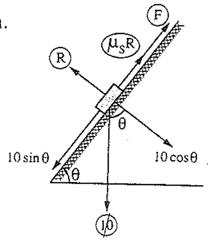
$\therefore F = \mu_s R + 10 \sin\theta = \frac{1}{2} R + 10 \times \frac{4}{5}$

$\therefore F = \frac{1}{2} R + 8$

, $R = 10 \cos\theta = 10 \times \frac{3}{5} = 6$

$\therefore F = \frac{1}{2} \times 6 + 8 = 11$ newton.

When the body is about to move downwards



$\therefore F + \mu_s R = 10 \sin\theta$

$\therefore F + \frac{1}{2} \times 6 = 10 \times \frac{4}{5}$

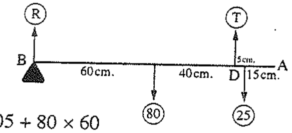
$\therefore F + 3 = 8$

F = 5 newton.

14 ⓑ **15** ⓑ **16** ⓐ

17

First :



$T - R = 105$

, $\because M_B = 0$

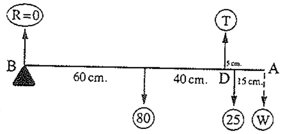
$\therefore T \times 100 = 25 \times 105 + 80 \times 60$

$\therefore T = 74.25$ gm.wt.

, $R = 105 - 74.25 = 30.75$ gm.wt.

Second :

Let the weight can be suspended at A which makes the rod about to seperate from the support = w



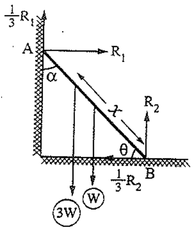
, $\because M_D = 0$

$\therefore w \times 20 + 25 \times 5 = 80 \times 40$

$\therefore w = 153.75$ gm.wt.

18

Let the measure of the angle between the ladder and the ground be θ and that with the wall be α



$\therefore \tan\theta = \cot\alpha = \frac{11}{6}$

The equations of equilibrium :

$R_1 = \frac{1}{3} R_2$, $R_2 + \frac{1}{3} R_1 = 4$ w

$\therefore 3 R_1 + \frac{1}{3} R_1 = 4$ w $\therefore \frac{10}{3} R_1 = 4$ w

$\therefore R_1 = \frac{6}{5}$ w $\therefore R_2 = \frac{18}{5}$ w

, M_B = zero

$\therefore w \times \frac{\ell}{2} \cos\theta + 3\, w\, x \cos\theta - R_1 \times \ell \sin\theta - \frac{1}{3} R_1\, \ell \cos\theta = 0$

Dividing by cos θ

$\therefore \frac{w\ell}{2} + 3\, w\, x - \frac{6}{5}\, w\, \ell \tan\theta - \frac{2}{5}\, w\, \ell =$ zero

$\therefore \frac{\ell}{2} + 3\, x - \frac{6}{5}\, \ell \times \frac{11}{6} - \frac{2}{5}\, \ell =$ zero

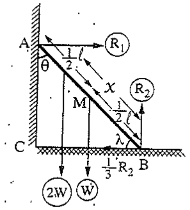
$\therefore 3\, x = \frac{21}{10}\, \ell$ $\therefore x = \frac{7}{10}\, \ell$

Model 17

1 ⓓ **2** ⓐ **3** ⓑ **4** ⓒ

5

$\tan\lambda = \cot\theta = 2$



The equation of equilibrium :

$R_1 = \frac{1}{3} R_2$

, $R_2 = 3$ w

$\therefore R_1 = \frac{1}{3} \times (3\text{ w}) = $ w

, $\because M_B = 0$

$\therefore R_1 \times \ell \sin\lambda = w \times \frac{1}{2} \times \ell \cos\lambda + 2\, w \times x \cos\lambda$ (divide by cos λ)

$\therefore R_1 \times \ell \tan\lambda = \frac{1}{2}\, w\, \ell + 2\, w\, x$

$w\, \ell \tan\lambda = \frac{1}{2}\, w\, \ell + 2\, w\, x$ (divide by w)

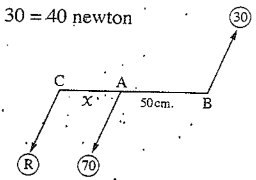
$\therefore \ell \tan\lambda = \frac{1}{2}\, \ell + 2\, x$ $\therefore 2\, \ell = \frac{1}{2}\, \ell + 2\, x$

$\therefore 2\, x = \frac{3}{2}\, \ell$ $\therefore x = \frac{3}{4}\, \ell$

$\therefore$ The point at a distance $\frac{3}{4}\, \ell$ from "B"

6

The resultant = R = 70 − 30 = 40 newton



$30 \times BC = 70 \times AC$

$30\, (50 + x) = 70 \times x$

$1500 + 30\, x = 70\, x$

$x = 37.5$ cm.

$\therefore$ The line of action of the resultant acts at a distance 37.5 cm. from the force 70 newton.

7 ⓐ

8

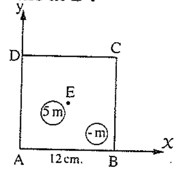
[a] $\because$ The masses are fixed at the vertices of the square are equal, and the fifth mass is fixed at the centre of the square

$\therefore$ The centre of the gravity of the system is the geometrical centre of the square

$\therefore$ Centre of gravity = (6 , 6)

*** Determining the centre of gravity of the system after removing the mass at B :**

Mass	E	B
	5 m	− m
x	6	12
y	6	0

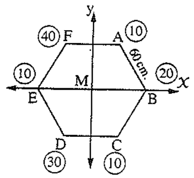


$\therefore x_G = \frac{5\,m \times 6 - m \times 12}{5\,m - m} = 4.5$

, $y_G = \frac{5\,m \times 6}{5\,m - m} = 7.5$

$\therefore G = (4.5 , 7.5)$

[b]



Consider $\overrightarrow{MB}$, $\overrightarrow{ME}$ two orthogonal directions.

	A	B	C	D	E	F
m	10	20	10	30	10	40
x	30	60	30	−30	−60	−30
y	$30\sqrt{3}$	0	$-30\sqrt{3}$	$-30\sqrt{3}$	0	$30\sqrt{3}$

$x_G = \frac{10 \times 30 + 20 \times 60 + 10 \times 30 + 30 \times -30 + 10 \times -60 + 40 \times -30}{10 + 20 + 10 + 30 + 10 + 40}$

$= -7.5$

, $y_G = \frac{10 \times 30\sqrt{3} + 10 \times -30\sqrt{3} + 30 \times 30\sqrt{3} + 40 \times 30\sqrt{3}}{10 + 20 + 10 + 30 + 10 + 40}$

$= 2.5\sqrt{3}$

$\therefore$ Centre of gravity = $(-7.5 , 2.5\sqrt{3})$

, distance between the centre of gravity and the centre of the hexagon $= \sqrt{(-7.5)^2 + (2.5\sqrt{3})^2}$

$= 5\sqrt{3}$ cm.

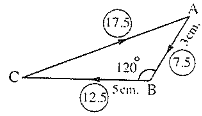
9 ⓒ **10** ⓑ

11

[a] $(AC)^2 = (3)^2 + (5)^2 - 2 \times 3 \times 5 \cos 120°$

$= 49$

$\therefore AC = 7$ cm.



$\therefore \frac{7.5}{3} = \frac{12.5}{5} = \frac{17.5}{7}$

$= 2.5$

, $\because$ the forces taken in the same cyclic order

$\therefore$ The system of forces is equivalent to a couple whose algebric measure

$= -2 \times$ area of Δ ABC × 2.5

$= -2 \times \frac{1}{2} \times 5 \times 3 \sin 120° \times 2.5$

$= \frac{-75\sqrt{3}}{4}$ newton. cm.

$\therefore$ Its norm $= \frac{75\sqrt{3}}{4}$ newton.cm.

[b] Let $\vec{F} = \ell\,\hat{i} + m\,\hat{j}$

$r_1 = \overrightarrow{BA} = \vec{A} - \vec{B}$

$= (-3 , 2) - (3 , 1) = (-6 , 1)$

, $M_B = (-6 , 1) \times (\ell , m) = 6\,m - \ell = 28$ (1)

, $r_2 = \overrightarrow{CA} = \vec{A} - \vec{C} = (-3 , 2) - (-1 , 4)$

$= (-2 , -2)$

, $\overrightarrow{M_C} = (-2 , -2) \times (\ell , m)$

$\therefore -2\,m + 2\,\ell = 28$

$\therefore -m + \ell = 14$ (2)

From (1) , (2) : $m = -6$, $\ell = 8$

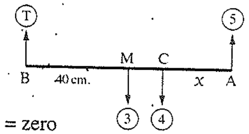
$\therefore \vec{F} = 8\,\hat{i} - 6\,\hat{j}$

12

From conditions of equilibrium :

$\therefore 5 + T = 7$

$\therefore T = 2$



M_A = zero

$\therefore 4\,x + 3 \times 40 - 2 \times 80 =$ zero

$\therefore x = 10$

$\therefore$ The weight should be hanged at a distance not less than 10 cm. from any end.

Another solution :

$\because T_1 + T_2 = 7$

, $\because$ the maximum tension in any of the two strings is 5 kg.wt.

$\therefore$ The least tension in the other one = 2 kg.wt.

$\therefore 2 \le$ the tension in any string ≤ 5

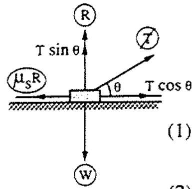
, $\because M_A = 0$ $\quad \therefore 4X + 3 \times 40 = T \times 80$

$\therefore T = \frac{4X + 120}{80} = \frac{X + 30}{20}$ $\quad \therefore 2 \le \frac{X + 30}{20} \le 5$

$\therefore 10 \le X \le 70$

i.e. The weight can be suspended at a distance between 10 cm. , 70 cm. from A or exactly at one of them.

13



$\mu_s = \tan \propto$

$\because$ The body is about to begin motion.

$\therefore T \cos \theta = \mu_s R = R \tan \propto$ (1)

$R + T \sin \theta = w$

$\therefore R = w - T \sin \theta$ (2)

From (1) and (2) :

$\therefore T \cos \theta = \tan \propto (w - T \sin \theta)$

$\therefore T \cos \theta = \frac{\sin \propto}{\cos \propto} (w - T \sin \theta)$

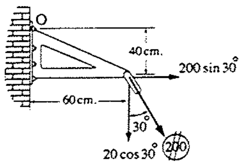
$\therefore T \cos \theta \cos \propto = w \sin \propto - T \sin \theta \sin \propto$

$\therefore T (\cos \theta \cos \propto + \sin \theta \sin \propto) = w \sin \propto$

$\therefore T \cos (\theta - \propto) = w \sin \propto$

$\therefore T = \frac{w \sin \propto}{\cos (\theta - \propto)} = w \sin \propto \sec (\theta - \propto)$

14



$M_o = 200 \sin 30° \times 40$

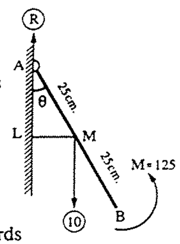
$- 200 \cos 30° \times 60$

$= 4000 - 6000\sqrt{3}$

$\simeq - 6392.3$ newton.cm.

15 ⓐ **16** ⓐ **17** ⓓ

18



$\because$ The rod is in equilibrium under the action of two couples

$\therefore$ The two forces (10 , R) form a couple , the algebraic measure of its moments = − 125 newton.cm.

$\therefore$ R = 10 newton vertically upwards

$\therefore - 10 \times ML = - 125$ $\quad \therefore ML = 12.5$ cm.

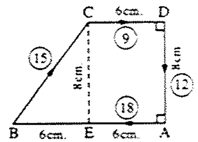
$\therefore \sin \theta = \frac{12.5}{25} = \frac{1}{2}$ $\quad \therefore \theta = 30°$ or $150°$

$\therefore$ The angle of inclination of the rod to the horizontal = 60° upwards or downwards.

Model 18

1 ⓑ **2** ⓒ **3** ⓐ **4** ⓒ

5



$CB = \sqrt{8^2 + 6^2} = 10$ cm.

$\because \frac{12}{8} = \frac{18}{12} = \frac{15}{10} = \frac{9}{6} = 1.5$

, $\because$ the forces act in one cyclic order

$\therefore$ The forces equivalent to a couple whose magnitude of its moment = 2 × area of the trapezium × 1.5

$= 2 \times \frac{1}{2} (12 + 6) \times 8 \times 1.5$

= 216 newton.cm.

6

[a] $\because \overrightarrow{F_1} // \overrightarrow{F_2}$ $\quad \therefore \frac{2}{-6} = \frac{a}{3}$ $\quad \therefore a = -1$

$\therefore \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} = -4\hat{i} + 2\hat{j}$

The moment of the resultant about origin = the sum of the moments of the two forces about the origin

$(X , 0) \times (-4 , 2)$

$= (1 , 0) \times (2 , -1) + (5 , 0) \times (-6 , 3)$

$\therefore 2X = -1 + 15 = 14$ $\quad \therefore X = 7$

$\therefore$ The line of action of the resultant intersects the X-axis at (7 , 0)

[b] $\overrightarrow{M_B} = -\overrightarrow{M_C}$

$\therefore$ The line of action of $\overrightarrow{F}$ bisects $\overline{BC}$.

$\therefore$ The line of action of $\overrightarrow{F}$ passes through

$D\left(\frac{1-1}{2} , \frac{-4+0}{2}\right) = (0 , -2)$

assume $\overrightarrow{F} = \ell\hat{i} + m\hat{j}$

$\therefore \overrightarrow{r_1} = \overrightarrow{AD} = \overrightarrow{D} - \overrightarrow{A} = (0 , -2) - (3 , 2) = (-3 , -4)$

, $\because \overrightarrow{M_A} = 60\hat{k}$

$\therefore 60\hat{k} = (-3 \times m + 4 \times \ell)\hat{k}$

$\therefore 4\ell - 3m = 60$ (1)

, $\overrightarrow{r_2} = \overrightarrow{BD} = \overrightarrow{D} - \overrightarrow{B} = (0 , -2) - (1 , -4) = (-1 , 2)$

, $\because \overrightarrow{M_B} = 60\hat{k}$

$\therefore 60\hat{k} = (-1 \times m - 2 \times \ell)\hat{k}$

$\therefore - m - 2\ell = 60$ (2)

By solving (1) , (2) : $\therefore m = -36$, $\ell = -12$

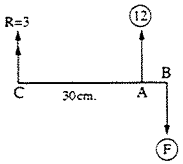
$\therefore \overrightarrow{F} = -12\hat{i} - 36\hat{j}$

$F = \sqrt{(-12)^2 + (-36)^2} = 12\sqrt{10}$ force unit.

7 ⓒ

8

[a] $\because R < 12$



$\therefore$ The two forces (12 , F) act on opposite directions and F < 12 or F > 12

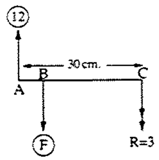
• If : F < 12

Then F = 12 − 3 = 9 newton

, 12 × 30 = 9 (30 + AB)

$\therefore$ AB = 10 cm.

$\therefore$ The distance between the two forces = 10 cm.



• If : F > 12

, then F = 12 + 3 = 15 newton

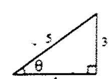
$\therefore$ 12 × 30 = 15 (30 − AB) $\therefore$ AB = 6 cm.

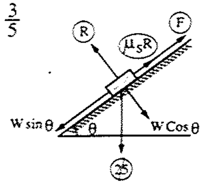
$\therefore$ The distance between the two forecs = 6 cm.

[b] $\because$ The body in equilibrium

$\therefore F + \mu_s R = w \sin \theta$

$\therefore F + \frac{1}{5} R = 25 \times \frac{3}{5}$





$\therefore F + \frac{1}{5} R = 15$ (1)

, $R = w \cos \theta = 25 \times \frac{4}{5} = 20$ (2)

, by substituting from (2) in (1) :

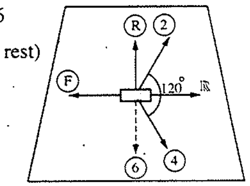
$\therefore F + \frac{1}{5} \times 20 = 15$ $\quad \therefore F + 4 = 15$

$\therefore$ F = 11 kg.wt.

9 ⓐ **10** ⓓ **11** ⓐ **12** ⓑ

13

$\mathbb{R} = \sqrt{4 + 16 + 2 \times 4 \times 2 \times -\frac{1}{2}} = 2\sqrt{3}$



, $\because$ the body at rest

$\therefore \mathbb{R} = F = 2\sqrt{3}$, R = 6

, $\because F \le \mu_s R$ (the body at rest)

$\therefore 2\sqrt{3} \le 6\mu_s$

$\therefore \mu_s \ge \frac{1}{\sqrt{3}}$

$\therefore \tan \lambda \ge \frac{1}{\sqrt{3}}$

$\therefore$ The measure of the friction angle must be not less than 30° "$\tan 30° = \frac{1}{\sqrt{3}}$"

, when the friction angle measure = 45°

$\therefore \tan \lambda = 1$ $\quad \therefore \mu_s = 1$

, when the force 2 newton becomes F

$\because$ The body is about to move

$\therefore \sqrt{16 + F^2 + 2 \times 4 \times F \times -\frac{1}{2}} = \mu_s R$

$\therefore \sqrt{F^2 - 4F + 16} = 1 \times 6$ by squaring both sides

$\therefore F^2 - 4F + 16 = 36$

$\therefore F^2 - 4F - 20 =$ zero

$\therefore F = \frac{4 \pm \sqrt{16 - 4 \times 1 \times -20}}{2 \times 1} = \frac{4 \pm \sqrt{96}}{2}$

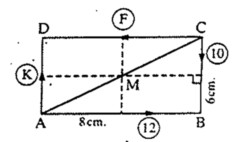
$\therefore F = \frac{4 \pm 4\sqrt{6}}{2} = 2 + 2\sqrt{6}$ newton and the other solution refused

The direction of motion in direction of $\mathbb{R}$ which inclined to the force (4) by an angle of measure θ

where $\tan \theta = \frac{(2 + 2\sqrt{6}) \sin 120°}{4 + (2 + 2\sqrt{6}) \cos 120°} = 3\sqrt{3} + 4\sqrt{2}$

$\therefore m(\angle \theta) \simeq 84° 44'$

14



$M_C = 0$

$12 \times 6 - k \times 8 = 0$

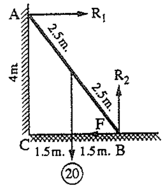
$\therefore k = 9$

, $M_M = 0$

$\therefore F \times 3 - k \times 4 + 12 \times 3 - 10 \times 4 = 0$

$\therefore F = \frac{40}{3}$

15



Let the ladder be in equilibrium

$R_1 = F$

$R_2 = 20$

, M_B = zero

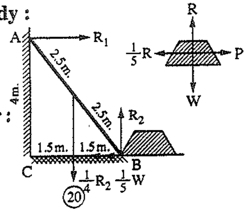
$20 \times 1.5 - R_1 \times 4$ = zero

$30 = 4 R_1$ $\therefore R_1 = 7\frac{1}{2}$ $\therefore F = 7\frac{1}{2}$

, $\because$ the magnitude of F_s at $B = \mu_s R_2 = \frac{1}{4} \times 20 = 5$

$\therefore$ The ladder can not be in equilibrium in this case because $F > F_s$

When the body of weight (w) is placed



*** With respect to the body :**

$\therefore P = \frac{1}{5} R$, $w = R$

$\therefore P = \frac{1}{5} w$

*** With respect to the ladder :**

$R_1 = \frac{1}{4} R_2 + \frac{1}{5} w$

$R_2 = 20$, M_B = zero

$20 \times 1.5 - R_1 \times 4$ = zero

$\therefore 30 = 4 R_1$ $\therefore R_1 = 7\frac{1}{2}$

$\therefore 7\frac{1}{2} = \frac{1}{4} \times 20 + \frac{1}{5} w$ $\therefore w = 12\frac{1}{2}$ kg.wt.

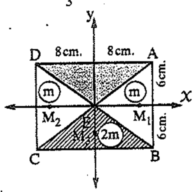
---

16 ⓒ

17

Let the triangle ABE mass be m and acts at M_1 and the triangle ECD be m and acts at M_2 , the triangle EBC be 2 m and acts at M_3

	M_1	M_2	M_3
Mass	m	m	2 m
x	$\frac{16}{3}$	$-\frac{16}{3}$	0
y	0	0	−4



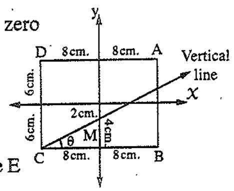
$$x_G = \frac{m \times \frac{16}{3} + m \times \frac{-16}{3}}{4\,m} = \text{zero}$$

$$, y_G = \frac{2\,m \times -4}{4\,m} = -2$$

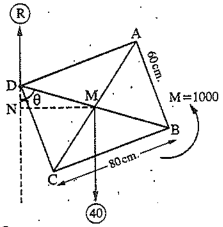
$\therefore$ The centre of gravity $= (0 , -2)$ with respect to the centre of the rectangle E

When suspending from C :

$\tan\theta = \frac{4}{8} = \frac{1}{2}$



18



$\because BD = \sqrt{60^2 + 80^2}$

$= 100$ cm.

MD = 50 cm.

$\therefore MN = 50 \sin\theta$

$\because$ The lamina is equilibrium under effect of two couples

$\therefore$ The two forces (R , 40) form a couple whose algebraic measure = − 1000 newton.cm.

$\therefore$ R = 40 newton , $-1000 = -40 \times MN$

$\therefore -1000 = -40 \times 50 \sin\theta$ $\therefore \sin\theta = \frac{1}{2}$

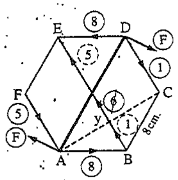
$\therefore m(\angle\theta) = 30°$ or $150°$

$\therefore$ The measure of inclination angle of $\overline{BD}$ to the vertical equals 30° or 150°

Model 19

1 ⓑ **2** ⓒ **3** ⓓ **4** ⓒ

5



$\because m(\angle BCY) = 30°$

$\therefore BY = \frac{1}{2} BC = 4$ cm.

, $CY = 4\sqrt{3}$ cm.

, $AE = 8\sqrt{3}$ cm.

Divide the 6 newton force which acts in direction $\overrightarrow{BE}$ into two forces 5 newton and 1 newton act in the same direction.

$\therefore$ The two forces (8 , 8) form a couple whose algebraic measure $= 8 \times 8\sqrt{3} = 64\sqrt{3}$

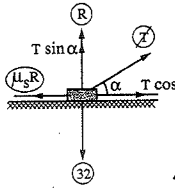
, the forces (1 , 1) form a couple whose algebraic measure $= -1 \times 4\sqrt{3} = -4\sqrt{3}$

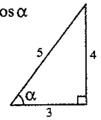
, the two forces (5 , 5) form a couple whose algebraic measure $= 5 \times 4\sqrt{3} = 20\sqrt{3}$.

$\therefore$ The system equivalent to a couple whose algebraic measure $= 80\sqrt{3}$ gm.wt.cm.

$\therefore$ The magnitude of couple moment $= 80\sqrt{3}$ gm.wt.cm.

6





$\mu_s = \tan\theta = \frac{1}{4}$

$R + T \sin\alpha = 32$

$\therefore R + \frac{4}{5} T = 32$ (1)

$T \cos\alpha = \mu_s R$

$\therefore \frac{3}{5} T = \frac{1}{4} R$

$\therefore R = \frac{12}{5} T$ (2)

By substituting from (2) in (1) :

$\therefore \frac{12}{5} T + \frac{4}{5} T = 32$

$\therefore$ T = 10 kg.wt.

By substituting in (2) :

$\therefore R = \frac{12}{5} \times 10 = 24$ kg.wt.

7

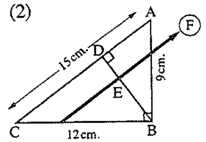
$AC = \sqrt{9^2 + 12^2} = 15$ cm.

$\therefore M_A = M_C = 36$ newton.cm.

$\therefore$ The line of action of $\overrightarrow{F}$ // $\overleftrightarrow{CA}$ and intersects $\overline{BD}$ at E such that $F \times DE = 36$ (1)

, $\because M_B = -72$ newton.cm.

$\therefore F \times BE = 72$ (2)



Divide (2) by (1) : $\therefore \frac{BE}{ED} = \frac{2}{1}$

, $\because BD = \frac{9 \times 12}{15} = 7.2$ cm.

$\therefore BE = \frac{2}{3} \times 7.2 = 4.8$ cm.

Substituting in (2) :

$\therefore F \times 4.8 = 72$ $\therefore$ F = 15 newton

$\therefore$ The magnitude of $\overrightarrow{F}$ is 15 newton and its line of action is parallel to $\overrightarrow{CA}$ and intersects $\overline{BD}$ of E where BE = 4.8 cm.

---

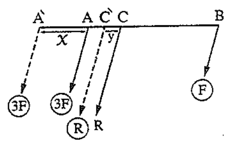
8 ⓐ **9** ⓐ **10** ⓐ

11

[a] $F \times BC = 3 F \times AC$

$\therefore BC = 3 AC$

When the force 3 F has been moved



$F \times B\grave{C} = 3 F \times A\grave{C}$

$BC + y = 3 (x + AC - y)$

$BC + y = 3x + 3AC - 3y$

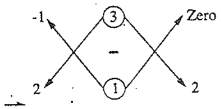
$\therefore 4y = 3x$

$y = \frac{3}{4} x$

[b] $\overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} = -6\hat{i} + 2\hat{j}$

$\therefore \overrightarrow{F_2} = -9\hat{i} + 3\hat{j}$

$= -3(3\hat{i} - \hat{j}) = -3\overrightarrow{F_1}$



$\therefore$ The two forces $\overrightarrow{F_1}$, $\overrightarrow{F_2}$ are parallel and act in opposite directions , assuming the resultant acts at C (x , y)

$\therefore$ C divides $\overline{AB}$ externally such that $\frac{AC}{CB} = \frac{3}{1}$

$\therefore x = \frac{3 \times 1 - 1 \times (-1)}{3 - 1} = 2$

$y = \frac{3 \times 2 - 1 \times 0}{3 - 1} = 3$ $\therefore C = (2 , 3)$

12

[a] The area $M_1 : M_2 : M$

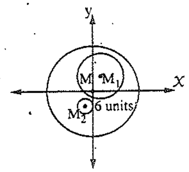
$= 9\pi : \pi : 36\pi$

$= 9 : 1 : 36$

Let the mass of M_1

$= 9$ m , M_2 = m , M_3 = 36 m

$\because M_1 = (1 , 2)$, $M_2 = (-1 , -3)$



Mass	− 9 m	− m	36 m
x	1	− 1	0
y	2	− 3	0

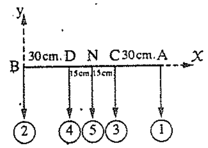
$$x_G = \frac{-9\,m \times 1 - m \times -1}{36\,m - 9\,m - m} = -\frac{4}{13}$$

$$y_G = \frac{-9\,m \times 2 - m \times -3}{36\,m - 9\,m - m} = -\frac{15}{26}$$

$\therefore$ The centre of gravity $= \left(-\frac{4}{13} , -\frac{15}{26}\right)$

[b] Consider $\overrightarrow{Bx}$, $\overrightarrow{By}$ are two orthogonal directions

	B	D	N	C	A
Mass	2	4	5	3	1
x	0	30	45	60	90



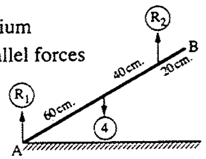
$X_G = \frac{4 \times 30 + 5 \times 45 + 3 \times 60 + 1 \times 90}{2 + 4 + 5 + 3 + 1} = 41$

i.e. the centre of gravity 41 cm. distant from B

i.e. at a distance 49 cm. from A

13

First : $\because$ The rod is in equilibrium under effect of 3 parallel forces



$\therefore R_1 + R_2 = 4$ (1)

$, R_1 \times 60 = R_2 \times 40$

$\therefore R_1 = \frac{2}{3} R_2$ (2)

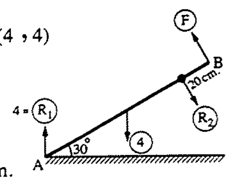
By substituting from (2) in (1) :

$\therefore \frac{2}{3} R_2 + R_2 = 4$ $\therefore \frac{5}{3} R_2 = 4$

$\therefore R_2 = \frac{12}{5} = 2.4$ kg.wt.

$\therefore R_1 = \frac{8}{5} = 1.6$ kg.wt.

Second : $\because$ The two forces (4 , 4) form a couple whose algebraic measure



$= -4 \times 60 \sin 60°$

$= -120\sqrt{3}$ kg.wt.cm.

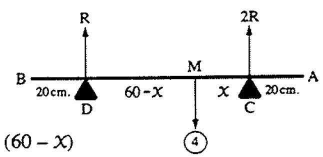
$\therefore$ The two forces (F , R_2) form a couple whose algebraic measure $= 120\sqrt{3}$ kg.wt.cm.

$\therefore F = R_2$ and act in the opposite direction

$\therefore F \times 20 = 120\sqrt{3}$ $\therefore F = 6\sqrt{3}$ kg.wt.

14

$3R = 3$



$\therefore R = \frac{4}{3}$

$, M_M = 0$

$\therefore 2R \times x = R \times (60 - x)$

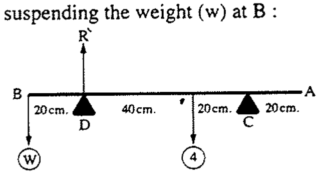
$\therefore 2x = 60 - x$

$\therefore 3x = 60$

$\therefore x = 20$ cm.

$\therefore$ The weight acts at a distance 40 cm. from A

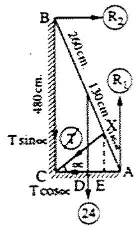
After suspending the weight (w) at B :



$\therefore w \times 20 = 4 \times 40$ $\therefore w = 8$ kg.wt.

15 ⓒ **16** ⓑ

17



$AC = \sqrt{(520)^2 - (480)^2} = 200$ cm.

$\therefore AD = DC = 100$ cm.

$\therefore AE = ED = 50$ cm.

$\therefore XE = \sqrt{(130)^2 - (50)^2} = 120$ cm.

$XC = \sqrt{(120)^2 + (150)^2} = 30\sqrt{41}$ cm.

$\therefore \sin \alpha = \frac{120}{30\sqrt{41}} = \frac{4}{\sqrt{41}}$

$\cos \alpha = \frac{150}{30\sqrt{41}} = \frac{5}{\sqrt{41}}$

Equilibrium equations : $R_1 = 24 + T \sin \alpha$

$\therefore R_1 = 24 + \frac{4}{\sqrt{41}} T$ (1)

$, R_2 = T \cos \alpha$

$\therefore R_2 = \frac{5}{\sqrt{41}} T$ (2)

$, M_C = 0$

$\therefore R_1 \times 200 = R_2 \times 480 + 24 \times 100$

$\therefore 5 R_1 = 12 R_2 + 60$ (3)

By substituting from (1) , (2) in (3) :

$\therefore 5\left(24 + \frac{4}{\sqrt{41}} T\right) = 12 \times \frac{5}{\sqrt{41}} T + 60$

$\therefore 120 + \frac{20}{\sqrt{41}} T = \frac{60}{\sqrt{41}} T + 60$ $\therefore \frac{40}{\sqrt{41}} T = 60$

$\therefore T = \frac{60\sqrt{41}}{40} = \frac{3\sqrt{41}}{2}$ kg.wt.

Substitute in (1) :

$\therefore R_1 = 24 + \frac{4}{\sqrt{41}} \times \frac{3\sqrt{41}}{2} = 30$ kg.wt.

Substitute in (2) : $R_2 = \frac{5}{\sqrt{41}} \times \frac{3\sqrt{41}}{2} = 7.5$ kg.wt.

18 ⓐ

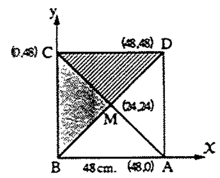
Model 20

1 ⓑ **2** ⓐ **3** ⓒ **4** ⓑ

5

[a] The area of the removed part Δ MCD :

The area of the square ABCD = 1 : 4



The triangle CMD centre

$= \left(\frac{24 + 48 + 0}{3} , \frac{24 + 48 + 48}{3}\right) = (24 , 40)$

The triangle CMB centre

$= \left(\frac{0 + 24 + 0}{3} , \frac{0 + 24 + 48}{3}\right) = (8 , 24)$

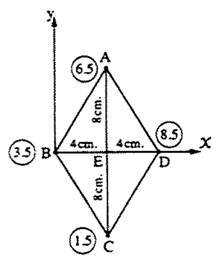
m	1	$-\frac{1}{4}$	$\frac{1}{4}$
x	24	24	8
y	24	40	24

$X_G = \frac{1 \times 24 - \frac{1}{4} \times 24 + \frac{1}{4} \times 8}{1 - \frac{1}{4} + \frac{1}{4}} = 20$ cm.

$y_G = \frac{1 \times 24 - \frac{1}{4} \times 40 + \frac{1}{4} \times 24}{1 - \frac{1}{4} + \frac{1}{4}} = 20$ cm.

$\therefore$ The lamina gravity centre is 20 cm. distant from $\overline{BA}$ and 20 cm. distant from $\overline{BC}$

[b] Consider $\overrightarrow{BX}$, $\overrightarrow{By}$ are two perpendicular directions



	A	B	C	D
Mass	6.5	3.5	1.5	85
x	4	0	4	8
y	8	0	-8	0

and so the coordinates of the centre of gravity of the system

$X_G = \frac{6.5 \times 4 + 1.5 \times 4 + 8.5 \times 8}{6.5 + 3.5 + 1.5 + 8.5} = 5$ cm.

$, y_G = \frac{6.5 \times 8 + 1.5 \times (-8)}{6.5 + 3.5 + 1.5 + 8.5} = 2$ cm.

$\therefore$ The centre of gravity (M) = (5 , 2)

$, \because E = (4 , 0)$

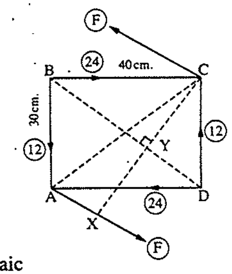
$\therefore ME = \sqrt{(5-4)^2 + (2)^2} = \sqrt{5}$ cm.

$\therefore$ The centre of gravity is $\sqrt{5}$ cm. distant from the rhombus centre.

6 ⓐ **7** ⓑ

8

$\because$ The two forces (24 , 24) form a couple whose algebraic measure



$= -24 \times 30$

$= -720$ kg.wt.cm.

, the two forces (12 , 12) form a couple whose algebraic measure $= 12 \times 40 = 480$ kg.wt.cm.

$\therefore$ The system equivalent to a couple whose algebraic measure $= -720 + 480 = -240$ kg.wt.cm.

$\therefore$ The magnitude of the couple moment = 240 kg.wt.cm.

$, \because BD = \sqrt{30^2 + 40^2} = 50$ cm.

$\therefore CY = \frac{30 \times 40}{50} = 24$ cm.

$\therefore CX = 2\,CY = 24 \times 2 = 48$ cm.

$\therefore$ the required two forces form a couple its moment = 240 kg.wt.cm.

$\therefore F \times CX = 240$

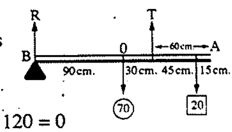
$\therefore F \times 48 = 240$

$\therefore F = 5$ kg.wt.

and act in direction $\overrightarrow{BD}$, $\overrightarrow{DB}$

9

From equilibrium equations



$M_B = 0$

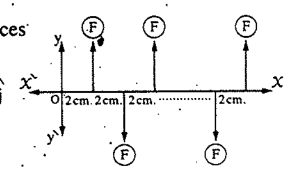
$-70 \times 90 - 20 \times 165 + T \times 120 = 0$

T = 80 newton.

10 ⓑ **11** ⓑ

12

$\because$ The number of forces = n "odd"



$\therefore \overrightarrow{M} = F\hat{j} - F\hat{j} + F\hat{j} - \cdots + F\hat{j}$

$\therefore \overrightarrow{M} = F \cdot \hat{j}$

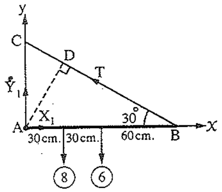
, $\because$ the point of action of the resultant is at a distance from $O = \frac{n+1}{2} \times 2 = (n+1)$ cm.

, $\because$ the algebraic sum of the moments about origin "O"

= the moment of the resultant about "O"

= F × (n + 1) = (n + 1) × F moment unit.

13



∵ The rod is in equilibrium

∴ X = zero

∴ $X_1 - T\cos 30°$ = zero

∴ $X_1 = \frac{\sqrt{3}}{2}T$ (1)

, y = zero

∴ $T\sin 30° + Y_1 - 6 - 8$ = zero

∴ $Y_1 = 14 - \frac{1}{2}T$ (2)

, M_A = zero

∴ $-8 \times 30 - 6 \times 60 + T \times AD$ = zero

Where $AD = AB \sin 30° = 120 \sin 30° = 60$

∴ $60\,T = 240 + 360 = 600$

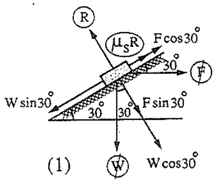
∴ T = 10 kg.wt.

From (1) , (2) : ∴ $X_1 = 5\sqrt{3}$, $Y_1 = 9$

∴ $R = \sqrt{(5\sqrt{3})^2 + (9)^2} = 2\sqrt{39}$ kg.wt.

14 ⓑ

15



[a] ∵ The body about to slide down

∴ $R = F\sin 30° + w\cos 30°$

∴ $R = \frac{1}{2}F + \frac{\sqrt{3}}{2}w$ (1)

$\mu_s R + F\cos 30° = w\sin 30°$

∴ $\frac{1}{3}R = \frac{1}{2}w - \frac{\sqrt{3}}{2}F$ (2)

Substitute from (1) in (2) :

∴ $\frac{1}{3}\left(\frac{1}{2}F + \frac{\sqrt{3}}{2}w\right) = \frac{1}{2}w - \frac{\sqrt{3}}{2}F$

∴ $\frac{1}{6}F + \frac{\sqrt{3}}{6}w = \frac{1}{2}w - \frac{\sqrt{3}}{2}F$

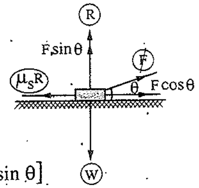
∴ $F + \sqrt{3}\,w = 3w - 3\sqrt{3}F$

∴ $(1 + 3\sqrt{3})F = (3 - \sqrt{3})w$

∴ $F = \frac{3-\sqrt{3}}{1+3\sqrt{3}}w = \frac{5\sqrt{3}-6}{13}w$

[b] $\mu_s = \tan\lambda$

, ∵ the body is about to move



∴ $F\cos\theta = \mu_s R$

∴ $F\cos\theta = R\tan\lambda$ (1)

, $R + F\sin\theta = w$

∴ $R = w - F\sin\theta$ (2)

From (1) , (2) :

∴ $F\cos\theta = \frac{\sin\lambda}{\cos\lambda}[w - F\sin\theta]$

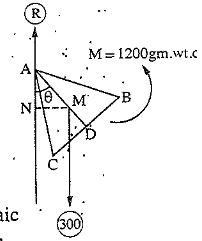
∴ $F\cos\theta\cos\lambda = w\sin\lambda - F\sin\theta\sin A$

∴ $F[\cos\theta\cos\lambda + \sin\theta\sin\lambda] = w\sin\lambda$

∴ $F[\cos(\lambda - 6)] = w.\sin\lambda$

∴ $F = \frac{w\sin\lambda}{\cos(\lambda-\theta)}$

16



∵ $AD = 8\sqrt{3}\sin 60° = 12$ cm.

∴ $AM = \frac{2}{3} \times 12 = 8$ cm.

, ∵ the lamina is in equilibrium under effect of couple and two forces (300 , R)

∴ The two forces (300 , R) form a couple whose algebraic measure is – 1200 gm.wt.cm.

∴ R = 300 gm.wt. , 300 × MN = 1200

∴ MN = 4 cm. ∴ $\sin\theta = \frac{4}{8} = \frac{1}{2}$

∴ m (∠ θ) = 30° or 150°

• If θ = 30° :

, ∵ m (∠ MAC) = 30°

∴ $\overline{AC}$ vertically downward

i.e. $\overline{AB}$ inclined to the horizontal at angle of measure 30°

• If θ = 150° :

, ∵ m (∠ DAB) = 30°

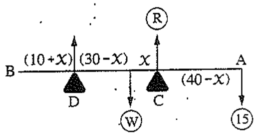
∴ $\overline{AB}$ vertically upward

i.e. inclined to the horizontal at angle of measure 90°

17 ⓑ

18

First case :



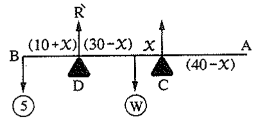
Let the rod weight = w kg.wt.

∵ The rod is about to overturn about C

∴ Reaction at D = zero

∴ $15 \times (40 - X) = W X$ (1)

Second case :



∵ The rod is about to overturn about D

∴ The reaction at C = 0

∴ $5(10 + X) = w(30 - X)$ (2)

Divide (1) by (2) :

∴ $\frac{3(40 - X)}{(10 + X)} = \frac{X}{30 - X}$

∴ $3(40 - X)(30 - X) = X(10 + X)$

∴ $3600 - 210X + 3X^2 = 10X + X^2$

∴ $2X^2 - 220X + 3600 = 0$

∴ $X^2 - 110X + 1800 = 0$

∴ $(X - 20)(X - 90) = 0$

∴ $X = 20$ or $X = 90$

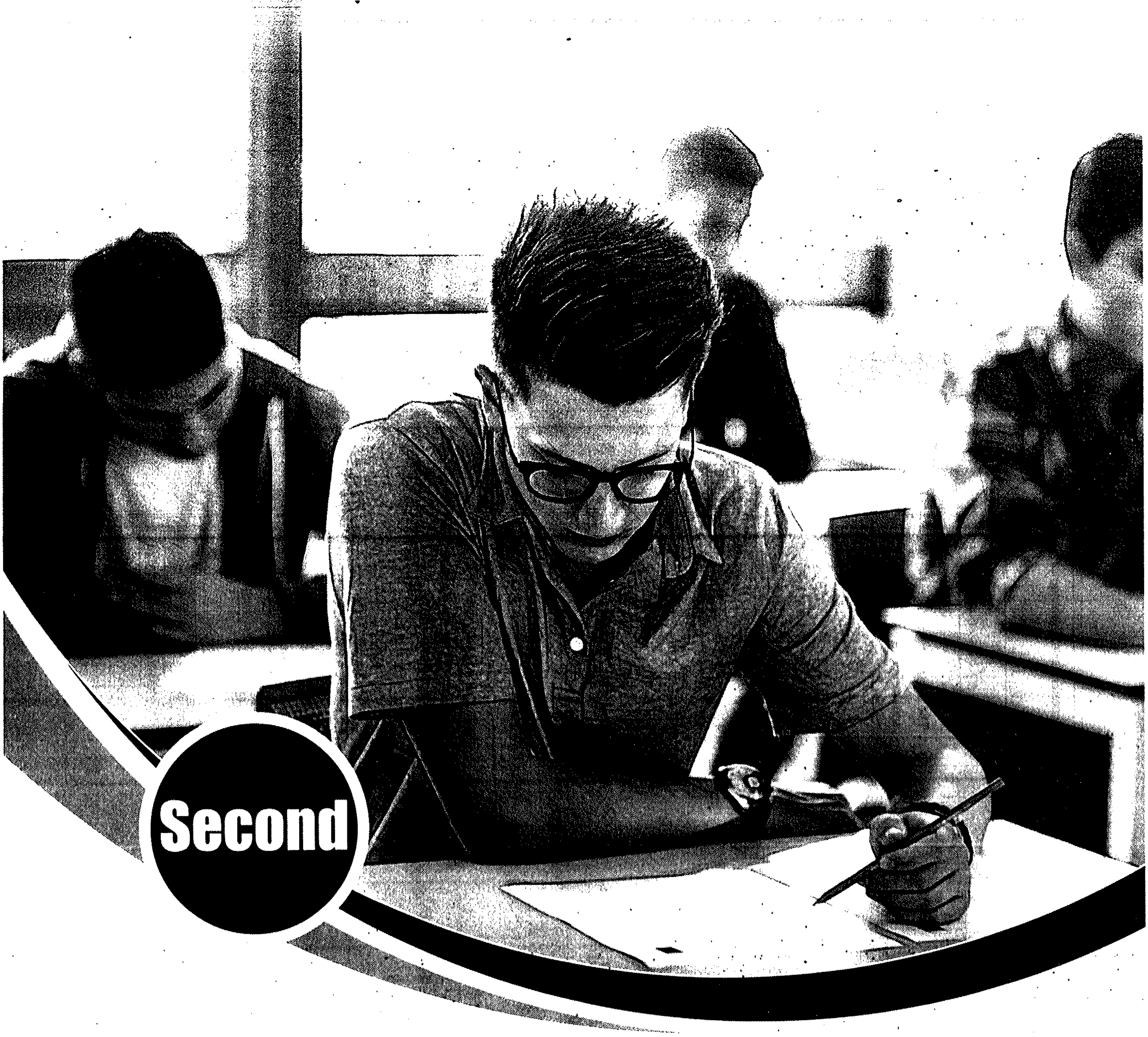
(Refused because the length of $\overline{AB}$ = 80 cm.)

∴ C is at a distance 20 cm. from A

, D is at a distance 30 cm. from B

By substituting in (1) : 15 × 20 = 20 × w

∴ w = 15 kg.wt.



Guide Answers of

School Book Examinations

Model 1

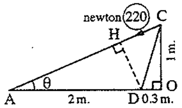
1

(1) (d)	(2) (b)	(3) (b)
(4) (c)	(5) (b)	(6) (b)

2

[a] $M_D = 220 \times DH$

$= 220 \times 2 \sin\theta$



, $\because AC = \sqrt{(1)^2 + (2.3)^2}$

$= \frac{\sqrt{629}}{10}$

$\therefore M_D = 220 \times 2 \times \frac{10}{\sqrt{629}}$

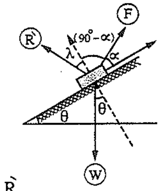
$\simeq 175.44$

$\therefore$ The magnitude of the moment

= 175.44 newton.m.

[b] $\because$ The body is about to move

$\therefore$ The body in equilibrium by using lami's rule :



$\frac{F}{\sin(180° - (\theta + \lambda))} = \frac{W}{\sin(90° - (\propto - \lambda))} = \frac{\hat{R}}{\sin(90° + (\propto + \theta))}$

$\therefore \frac{F}{\sin(\theta + \lambda)} = \frac{W}{\cos(\propto - \lambda)}$ $\therefore F = \frac{W \sin(\theta + \lambda)}{\cos(\propto - \lambda)}$

, $\vec{F}$ will be minimum value when $\cos(\propto - \lambda)$ is maximum

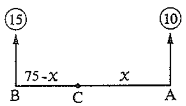
i.e. $\cos(\propto - \lambda) = 1$ $\therefore \propto - \lambda = 0$ $\therefore \propto = \lambda$

$\therefore$ The minimum value of $F = W \sin(\theta + \lambda)$ and makes an angle of measure λ with the plane upwards.

3

[a] $\mathbb{R} = 10 + 15 = 25$

In the same direction of the two forces.



, $\because 10\,X = 15\,(75 - X)$

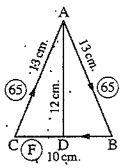
$\therefore 2\,X = 3\,(75 - X)$

$\therefore 5\,X = 225$ $\therefore X = 45$ cm.

i.e. The line of action of the resultant at a distance 45 cm. from A

[b] $\because$ The forces form a couple

, $\because$ the forces take the same way round , then :



$\frac{65}{13} = \frac{F}{10}$

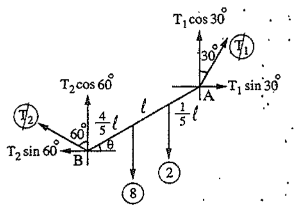
$\therefore F = 50$ newton

$\therefore$ The norm of the couple =

$2 \times \left(\frac{1}{2} \times 10 \times 12\right) \times \frac{65}{13} = 600$ newton.cm.

4

[a]



The equations of equilibrium

$T_1 \sin 30° = T_2 \sin 60°$

$\therefore \frac{1}{2} T_1 = \frac{\sqrt{3}}{2} T_2$ $\therefore T_1 = \sqrt{3}\, T_2$ (1)

, $T_1 \cos 30° + T_2 \cos 60° = 10$

$\therefore \frac{\sqrt{3}}{2} T_1 + \frac{1}{2} T_2 = 10$ $\therefore \sqrt{3}\, T_1 + T_2 = 20$ (2)

From (1) , (2) :

$\therefore T_1 = 5\sqrt{3}$ newton , $T_2 = 5$ newton.

, $M_B = 0$

$\therefore -8 \times \frac{4}{5} \ell \cos\theta - 2 \times \frac{9}{5} \ell \cos\theta - 5\sqrt{3} \sin 30° \times 2\ell \sin\theta + 5\sqrt{3} \cos 30° \times 2\ell \cos\theta = 0$

(dividing by $\ell \cos\theta$)

$\therefore -\frac{32}{5} - \frac{18}{5} - 5\sqrt{3} \tan\theta + 15 = 0$

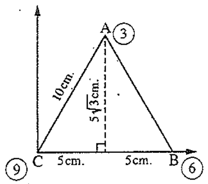
$\therefore 5\sqrt{3} \tan\theta = 5$ $\therefore \tan\theta = \frac{\sqrt{3}}{3}$

$\therefore m(\angle\theta) = 30°$

i.e. The rod inclined 30° with the horizontal.

[b]

	A	B	C
Mass	3	6	9
X	5	10	0
y	$5\sqrt{3}$	0	0



The coordinates of the centre of gravity of the system are :

$X_G = \frac{3 \times 5 + 6 \times 10 + 9 \times 0}{18} = 4\frac{1}{6}$

$y_G = \frac{3 \times 5\sqrt{3} + 6 \times 0 + 9 \times 0}{18} = \frac{5\sqrt{3}}{6}$

$\therefore$ The centre of gravity $= \left(4\frac{1}{6}, \frac{5\sqrt{3}}{6}\right)$ with respect to "C"

5

[a] From the figure :

$E = (0, 0, 10)$, $M = (5, 10, 5)$

$\therefore \overrightarrow{EM} = \vec{M} - \vec{E} = (5, 10, -5)$

$\therefore \vec{F} = F \times \frac{\overrightarrow{EM}}{\|\overrightarrow{EM}\|} = 25\sqrt{6} \times \frac{(5, 10, -5)}{\sqrt{25 + 100 + 25}}$

$= 5\,(5, 10, -5) = (25, 50, -25)$

$\therefore \overrightarrow{M_o} = \overrightarrow{OE} \times \vec{F}$

$= (0, 0, 10) \times (25, 50, -25)$

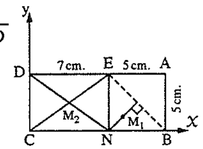
$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 10 \\ 25 & 50 & -25 \end{vmatrix} = -500\,\hat{i} + 250\,\hat{j}$

The component of the moment of the force $\vec{F}$ in direction of X-axis = − 500

, the component of the moment of the force $\vec{F}$ in direction of y-axis = 250

, the component of the moment of the force $\vec{F}$ in direction of z-axis = 0

[b] $\frac{\text{The area of square ABNE}}{\text{The area of rectangle ENCD}}$



$= \frac{5 \times 5}{5 \times 7} = \frac{5}{7}$

, let the mass of Δ BEN $= 5$ m at M_1

, the mass of rectangle ENCD = 7 m at M_2

$M_1 = \left(\frac{12 + 7 + 7}{3}, \frac{0 + 0 + 5}{3}\right) = \left(\frac{26}{3}, \frac{5}{3}\right)$

, $M_2 = (3.5, 2.5)$

	M_1	M_2
Mass	5 m	7 m
X	$\frac{26}{3}$	3.5
y	$\frac{5}{3}$	2.5

$\therefore X_G = \frac{5\,m \times \frac{26}{3} + 7\,m \times 3.5}{12\,m} = \frac{407}{72}$

, $y_G = \frac{5\,m \times \frac{5}{3} + 7\,m \times 2.5}{12\,m} = \frac{155}{72}$

i.e. The centre of gravity $= \left(\frac{407}{72}, \frac{155}{72}\right)$

Model 2

1

(1) (a)	(2) (a)	(3) (b)
(4) (b)	(5) (b)	(6) (b)

2

[a] $\overrightarrow{M_o} = \overrightarrow{OA} \times \vec{F} = (-1, 2, 1) \times (5, -1, 3)$

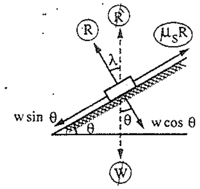
$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ 5 & -1 & 3 \end{vmatrix} = 7\hat{i} + 8\hat{j} - 9\hat{k}$

, the length of perpendicular from "O" to the line of action of $\vec{F}$

$= \frac{\|\overrightarrow{M_o}\|}{\|\vec{F}\|} = \frac{\sqrt{(7)^2 + (8)^2 + (-9)^2}}{\sqrt{(5)^2 + (-1)^2 + (3)^2}} = \frac{\sqrt{194}}{\sqrt{35}}$

$\simeq 2.35$ length unit.

[b] $\because$ The body about to move downwards



$\therefore R = W \cos\theta$ (1)

, $\mu_s R = W \sin\theta$ (2)

divide (2) ÷ (1) :

$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta$

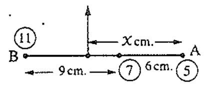
$\because \mu_s = \tan\lambda$ where λ is the angle of friction.

$\therefore \tan\lambda = \tan\theta$ $\therefore \lambda = \theta$

$\therefore$ The measure of the angle of friction = the measure of the angle of inclination of the plane to the horizontal.

3

[a] Let the suspended point at a distance X cm. from "A"



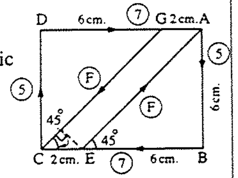
$\because$ The sum of the moment of the forces about A = the moment of resultant about A

$\therefore 7 \times 6 + 11 \times 15 = (5 + 7 + 11) \times X$

$\therefore X = 9$ cm.

i.e. The suspended point at a distance 9 cm. from A

[b] The two forces (5 , 5) form a couple its algebric measure (M_1) where :



$M_1 = -5 \times 8$

$= -40$ kg.wt.cm.

, the two forces (7 , 7) form a couple its algebric measure (M_2) where :

$M_2 = -7 \times 6 = -42$ kg.wt.cm.

, the two forces (F , F) form a couple its algebric measure $(M_3) = F \times 2 \sin 45° = \sqrt{2}\, F$

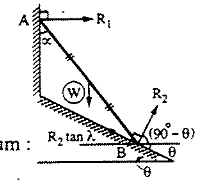
$, \because M = M_1 + M_2 + M_3$

$\therefore -40 - 42 + \sqrt{2}\, F = 10$

$\therefore F = \frac{92}{\sqrt{2}} = 46\sqrt{2}$ kg.wt.

4

[a] Let the ladder make an angle of measure $\propto$ with the vertical



The equations of equilibrium :

$R_1 + R_2 \cos (90° - \theta)$

$= R_2 \tan \lambda \cos \theta$

$\therefore R_1 + R_2 \sin \theta = R_2 \tan \lambda \cos \theta$

$\therefore R_1 = R_2 (\tan \lambda \cos \theta - \sin \theta)$

$= R_2 \left(\frac{\sin \lambda}{\cos \lambda} \cos \theta - \sin \theta\right)$

$= R_2 \left(\frac{\sin \lambda \cos \theta - \sin \theta \cos \lambda}{\cos \lambda}\right)$

$\therefore R_1 = R_2 \left(\frac{\sin (\lambda - \theta)}{\cos \lambda}\right)$ (1)

$, R_2 \sin (90° - \theta) + R_2 \tan \lambda \sin \theta = W$

$\therefore R_2 \cos \theta + R_2 \tan \lambda \sin \theta = W$

$\therefore R_2 (\cos \theta + \tan \lambda \sin \theta) = W$

$\therefore R_2 \left(\cos \theta + \frac{\sin \lambda}{\cos \lambda} \sin \theta\right) = W$

$\therefore R_2 (\cos \theta \cos \lambda + \sin \lambda \sin \theta) = W \cos \lambda$

$\therefore R_2 \cos (\lambda - \theta) = W \cos \lambda$

$\therefore R_2 = \frac{W \cos \lambda}{\cos (\lambda - \theta)}$ (2)

From (1) and (2) :

$\therefore R_1 = \frac{W \cos \lambda}{(\cos \lambda - \theta)} \times \frac{\sin (\lambda - \theta)}{\cos \lambda}$

$R_1 = W \tan (\lambda - \theta)$ (3)

$, M_B = 0$

$\therefore W \times \frac{l}{2} \sin \propto - R_1 \times l \cos \propto =$ zero

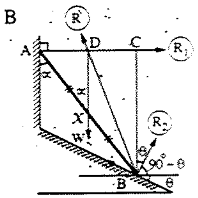
$\therefore \frac{W}{2} \tan \propto = R_1$ and from (3) :

$\therefore \frac{W}{2} \tan \propto = W \tan (\lambda - \theta)$

$\therefore \tan \propto = 2 \tan (\lambda - \theta)$

Another solution :

let the resultant reaction at B is ($\vec{R}$) and the weight and the reaction of the wall intersecting at D



From the figure :

$AD = DC$, $DX = \frac{1}{2}$ BC

, m ($\angle$ CBD) = $\lambda - \theta$ "λ is the angle between normal and resultant reactions"

$\therefore \tan (\lambda - \theta) = \frac{DC}{CB}$ (1)

In Δ ADX : $\tan \theta = \frac{AD}{DX} = \frac{DC}{\frac{1}{2} CB}$ (2)

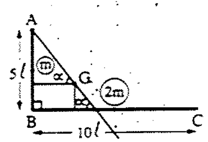
From (1) , (2) :

$\therefore \tan \theta = 2 \tan (\lambda - \theta)$

[b]

Let the angle of inclination of $\overline{BC}$ to the vertical is $\propto$

Mass	2 m	m
x	5 l	0
y	0	2.5 l



$X_G = \frac{2 m \times 5 l}{3 m} = \frac{10}{3} l$

$, y_G = \frac{m \times 2.5 l}{3 m} = \frac{5}{6} l$

$\therefore$ Centre of gravity $= \left(\frac{10}{3} l , \frac{5}{6} l\right)$

$\therefore \tan \propto = \frac{5 l - \frac{5}{6} l}{\frac{10}{3} l} = \frac{5}{4}$

$\therefore \overline{BC}$ inclines to the horizontal by an angle θ its tangent $= \frac{4}{5}$

5

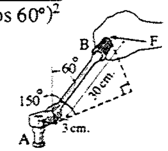
[a] $\because$ The length of perpendicular $= \frac{\| \overline{M_A} \|}{\| \overline{F} \|}$

, $\because$ the length of perpendicular

$= \sqrt{(3 + 30 \sin 60°)^2 + (30 \cos 60°)^2}$

$= 32.63$

$\therefore F = \frac{620}{32.63} \simeq 19$ newton



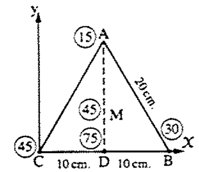
Another solution :

To get the length perpendicular :

$AB = \sqrt{(30)^2 + (3)^2 - 2 (30)(3) \cos 150°} \simeq 32.63$

[b] * **Centre of gravity of the system :**

Consider $\overrightarrow{Cx}$, $\overrightarrow{Cy}$ two orthogonal directions



Mass	45	75	30	15	45
x	0	10	20	10	10
y	0	0	0	$10\sqrt{3}$	$\frac{10\sqrt{3}}{3}$

$\therefore X_G = \frac{75 \times 10 + 30 \times 20 + 15 \times 10 + 45 \times 10}{45 + 75 + 30 + 15 + 45} = \frac{65}{7}$

$, y_G = \frac{15 \times 10\sqrt{3} + 45 \times \frac{10\sqrt{3}}{3}}{45 + 75 + 30 + 15 + 45} = \frac{10\sqrt{3}}{7}$

$\therefore G = \left(\frac{65}{7} , \frac{10\sqrt{3}}{7}\right)$

* **After removing the mass at B :**

Mass	210	− 30
x	$\frac{65}{7}$	20
y	$\frac{10\sqrt{3}}{7}$	0

$\therefore X_{\acute{G}} = \frac{210 \times \frac{65}{7} - 30 \times 20}{210 - 30} = \frac{15}{2}$

$, y_{\acute{G}} = \frac{210 \times \frac{10\sqrt{3}}{7}}{210 - 30} = \frac{5\sqrt{3}}{3}$

$\therefore \acute{G} = \left(\frac{15}{2} , \frac{5\sqrt{3}}{3}\right)$ From point "C"

Model 3

1

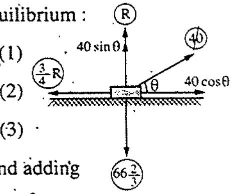
(1) 75 (2) 350 newton cm.

(3) $\pm 4 (\hat{i} - 2\hat{j})$ (4) 3000 newton cm.

(5) by centre of the sphere (6) its centroid

2

[a] $\because$ The equations of equilibrium :



$40 \cos \theta = \frac{3}{4} R$ (1)

$, R + 40 \sin \theta = 66\frac{2}{3}$ (2)

$\therefore 40 \sin \theta = 66\frac{2}{3} - R$ (3)

by squaring (1) , (3) and adding

$\therefore (40 \cos \theta)^2 + (40 \sin \theta)^2$

$= \left(\frac{3}{4} R\right)^2 + \left(66\frac{2}{3} - R\right)^2$

$\therefore 1600 (\cos^2 \theta + \sin^2 \theta)$

$= \frac{9}{16} R^2 + \frac{40000}{9} - \frac{400}{3} R + R^2$

$\therefore \frac{25}{16} R^2 - \frac{400}{3} R + \frac{40000}{9} = 1600$

$\therefore \frac{25}{16} R^2 - \frac{400}{3} R + \frac{25600}{9} = 0$

$\therefore R = \frac{128}{3}$

by substitute in equation (1) :

$\therefore \cos \theta = \frac{3}{160} \times \frac{128}{3} = \frac{4}{5}$ then $\theta\ 36° 52'$

[b] $\because M_o = 200 \sin 30° \times 40 - 200 \cos 30° \times 60$

$= -6392.3$ newton.cm.

3

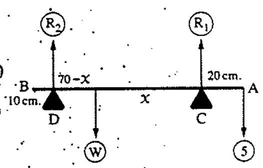
[a] As suspending the weight 5 kg.wt. at A , then the rod is about to rotate about C

$\therefore R_2 =$ zero

$, M_C =$ zero

$\therefore WX - 5 \times 20 = 0$

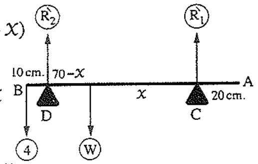
$\therefore WX = 100$ (1)



As suspending the weight 4 kg.wt. at B , then the rod is about to rotate about D

$\therefore \hat{R}_1 = \text{zero}$, $M_D = \text{zero}$

$\therefore 4 \times 10 - W(70 - x) = \text{zero}$



$\therefore 40 - 70\,W + W\,x = \text{zero}$ (2)

Substituting from (1) :

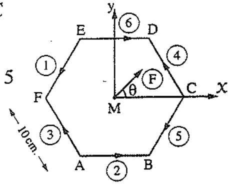
$\therefore 40 + 100 = 70\,W$

$\therefore W = 2$ kg.wt. and from (1) :

$x = \frac{100}{2} = 50$ cm.

$\therefore$ The weight of the rod = 2 kg.wt. acting at a distance 70 cm. from A

[b] Let $\hat{i}$, $\hat{j}$ be two unit vectors in the direction of $\overrightarrow{MC}$ and the perpendicular to it and the forces 2 , 5 , 4 , 6 , 1 , 3 newton are F_1 , F_2 , F_3 , F_4 , F_5 , F_6 respectively



$\therefore \overrightarrow{F_1} = 2\hat{i}$, $\overrightarrow{F_2} = 5\cos 60° (-\hat{i}) + 5 \sin 60° (-\hat{j})$

$= \frac{-5}{2}\hat{i} - \frac{5\sqrt{3}}{2}\hat{j}$

, $\overrightarrow{F_3} = 4\cos 60° (-\hat{i}) + 4\sin 60° \hat{j}$

$= -2\hat{i} + 2\sqrt{3}\hat{j}$

, $\overrightarrow{F_4} = 6\hat{i}$, $\overrightarrow{F_5} = 1\cos 60° (-\hat{i}) + 1 \sin 60° (-\hat{j})$

$= -\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$

, $\overrightarrow{F_6} = 3\cos 60° (-\hat{i}) + 3\sin 60° (\hat{j})$

$= \frac{-3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$\therefore \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \overrightarrow{F_4} + \overrightarrow{F_5} + \overrightarrow{F_6} = \frac{3}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$

, $\because$ the system tends to a couple

$\therefore \overrightarrow{F} + \overrightarrow{R} = \overrightarrow{O}$ $\quad \therefore \overrightarrow{F} = -\overrightarrow{R} = \frac{-3}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$

$\therefore \|\overrightarrow{F}\| = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}$ newton.

, $\because \tan\theta = \frac{y}{x} = \frac{1}{\sqrt{3}}$, $x < 0$, $y < 0$

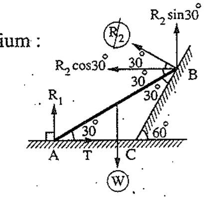
$\therefore \theta = 180° + 30° = 210°$

, $M_M = (2 - 5 + 4 - 6 + 1 - 3) \times 5\sqrt{3}$

$= -35\sqrt{3}$ newton cm.

4

[a] The equations of equilibrium :



$T = R_2 \cos 30°$

$T = \frac{\sqrt{3}}{2} R_2$ (1)

, $R_1 + \frac{1}{2} R_2 = W$ (2)

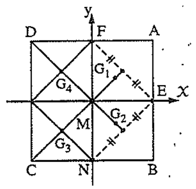
, $M_A = \text{zero}$

$\therefore -W \times \frac{\ell}{2}\cos 30° + R_2 \times \ell \sin 60° = 0$

$\therefore R_2 = \frac{W}{2}$ $\qquad \therefore R_1 + \frac{1}{2} \times \frac{W}{2} = W$

$\therefore R_1 = \frac{3}{4} W$, $T = \frac{\sqrt{3}}{2} \times \frac{W}{2} = \frac{\sqrt{3}}{4} W$

[b] Consider $\overrightarrow{ME}$, $\overrightarrow{MF}$ are two orthogonal directions.



	G_1	G_2	G_3	G_4
Mass	m	m	m	m
x	$\frac{\ell}{6}$	$\frac{\ell}{6}$	$\frac{-\ell}{4}$	$\frac{-\ell}{4}$
y	$\frac{\ell}{6}$	$\frac{-\ell}{6}$	$\frac{-\ell}{4}$	$\frac{\ell}{4}$

$\therefore x_G = \frac{m \times \frac{1}{6}\ell + m \times \frac{1}{6}\ell + m \times -\frac{1}{4}\ell + m \times -\frac{1}{4}\ell}{4\,m}$

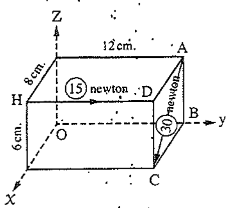
$= \frac{-1}{24}\ell$

, $y_G = \frac{m \times \frac{1}{6}\ell + m \times -\frac{1}{6}\ell + m \times -\frac{1}{4}\ell + m \times \frac{1}{4}\ell}{4\,m} = 0$

$\therefore$ Centre of gravity $= \left(-\frac{1}{24}\ell , 0\right)$ with respect to the centre of the square M

5

[a] From the figure



$\overrightarrow{HD} = \overrightarrow{D} - \overrightarrow{H}$

$= (8 , 12 , 6)$

$- (8 , 0 , 6)$

$= (0 , 12 , 0)$

$\therefore \overrightarrow{F_1} = 15 \times \frac{\overrightarrow{HD}}{\|\overrightarrow{HD}\|}$

$= 15 \times \frac{(0 , 12 , 0)}{\sqrt{(0)^2 + (12)^2 + (0)^2}} = (0 , 15 , 0)$

, $\overrightarrow{AC} = \overrightarrow{C} - \overrightarrow{A} = (8 , 12 , 0) - (0 , 12 , 6)$

$= (8 , 0 , -6)$

$\therefore \overrightarrow{F_2} = 30 \times \frac{\overrightarrow{AC}}{\|\overrightarrow{AC}\|} = 30 \times \frac{(8 , 0 , -6)}{\sqrt{(8)^2 + (0)^2 + (-6)^2}}$

$= (24 , 0 , -18)$

$\therefore \overrightarrow{M_O} = \overrightarrow{OA} \times \overrightarrow{F_2} + \overrightarrow{OH} \times \overrightarrow{F_1}$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 12 & 6 \\ 24 & 0 & -18 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 0 & 6 \\ 0 & 15 & 0 \end{vmatrix}$

$= -216\hat{i} + 144\hat{j} - 288\hat{k} - 90\hat{i} + 120\hat{k}$

$= -306\hat{i} + 144\hat{j} - 168\hat{k}$

[b] $x_G = \frac{20 \times 2 + 15 \times -3 + 25 \times 1}{20 + 15 + 25} = \frac{1}{3}$

, $y_G = \frac{20 \times 1 + 15 \times 1 + 25 \times -1}{20 + 15 + 25} = \frac{1}{3}$

$\therefore$ Centre of gravity is $\left(\frac{1}{3} , \frac{1}{6}\right)$

Model 4

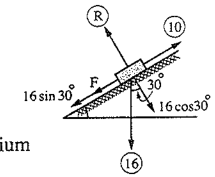
1

(1) (c)	(2) (b)	(3) (b)
(4) (c)	(5) (d) , (b)	(6) (a)

2

[a] $\because 10 > 16 \sin 30°$

$\therefore$ Force friction (F) acts downwards



, $\because$ the body in equilibrium

$\therefore F = 2$ kg.wt.

, $\because \mu R = \frac{1}{\sqrt{3}} \times 16 \cos 30° = 8$ kg.wt.

$\therefore F < \mu R$

$\therefore$ The body is in equilibrium but not about to move.

[b] $\because M_A = -150 \times 60 - 250 \times \frac{4}{5} \times 110 - 200 \cos 30° \times 150 + 200 \sin 30° \times 2$

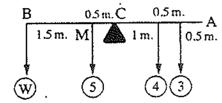
$= -56780.8$ kg.wt.cm.

, $M_B = 200 \sin 30° \times 2 + 250 \times \frac{4}{5} \times 40 + 150 \times 90$

$= 21700$ kg.wt.cm.

3

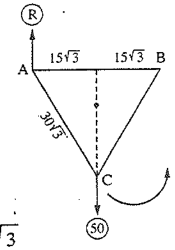
[a] $\because M_C = 0$



$\therefore 3 \times 1.5 + 4 \times 1 = 5 \times \frac{1}{2} + W \times 2$

$\therefore W = 3$ kg.wt.

[b] $\because$ The lamina is in equilibrium under the action of two couples



$\therefore$ (50 , R) form a couple

, the algebraic measure of its moment (M)

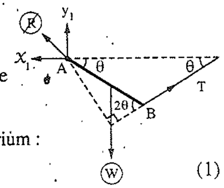
$\therefore R = 50$ gm.wt.

, $M = -50 \times 15\sqrt{3} = -750\sqrt{3}$

$\therefore$ The norm of the moment of the acting couple $= 750\sqrt{3}$ gm.wt.cm.

4

[a] Let the components of the reaction of the hinge be x_1 and y_1



The equations of equilibrium :

$x_1 = T \cos\theta$ (1)

$y_1 = W - T\sin\theta$ (2)

, $M_A = \text{zero}$

$\therefore T \times \ell \sin 2\theta = W \times \frac{\ell}{2} \cos\theta$

$\therefore T \times 2 \sin\theta \cos\theta = \frac{1}{2} W \cos\theta$

i.e. $T = \frac{1}{4} W \csc\theta$

Substituting in (1) : $\therefore x_1 = \frac{1}{4} W \cot\theta$

Substituting in (2) : $\therefore y_1 = W - \frac{1}{4} W = \frac{3}{4} W$

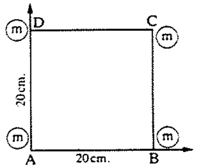
$\therefore$ R (the reaction of the hinge) $= \sqrt{x_1^2 + y_1^2}$

$\therefore R = \sqrt{\frac{1}{16} W^2 \cot^2\theta + \frac{9}{16} W^2}$

$= \frac{1}{4} W \sqrt{\cot^2\theta + 9}$

$= \frac{1}{4} W \sqrt{(\csc^2\theta - 1) + 9} = \frac{1}{4} W \sqrt{8 + \csc^2\theta}$

[b] (1) ∵ The four masses are equal



∴ The centre of gravity acts at the geometrical centre of the square (10 , 10)

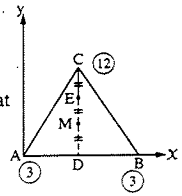
(2) After removing one of the masses , suppose it the mass at "C"

∴ The centre of gravity lies at the geometrical centre of the triangle ABD

i.e. At the centroid of the triangle $\left(\frac{20}{3}, \frac{20}{3}\right)$

5

[a] Distributing the mass 3 kg. among the vertices A , B , C to make the mass at A = 3 kg.



, at B = 3 kg.

, at C = 12 kg.

, consider $\overrightarrow{AX}$, $\overrightarrow{Ay}$ two orthogonal directions , let the side length of the triangle be ℓ

$\therefore CD = \frac{\sqrt{3}}{2}\ell$ $\quad\quad \therefore M = \left(\frac{1}{2}\ell, \frac{\sqrt{3}}{6}\ell\right)$

	A	B	C
m	3	3	12
x	0	ℓ	$\frac{1}{2}\ell$
y	0	0	$\frac{\sqrt{3}}{2}\ell$

$\therefore x_G = \frac{3 \times \ell + 12 \times \frac{1}{2}\ell}{3+3+12} = \frac{1}{2}\ell$

$, y_G = \frac{12 \times \frac{\sqrt{3}}{2}\ell}{3+3+12} = \frac{\sqrt{3}}{3}\ell$

∴ Centre of gravity of the system = $\left(\frac{1}{2}\ell, \frac{\sqrt{3}}{3}\ell\right)$

, ∵ E (midpoint of $\overline{CM}$) = $\left(\frac{1}{2}\ell, \frac{\sqrt{3}}{3}\ell\right)$

∴ The centre of gravity is the midpoint of $\overline{MC}$

[b] From the figure : A = (0 , 0 , 10)

, D = (6 , 8 , 0)

$\therefore \overrightarrow{AD} = \overrightarrow{D} - \overrightarrow{A} = (6, 8, -10)$

$\therefore \overrightarrow{F} = F \times \frac{\overrightarrow{AD}}{\|\overrightarrow{AD}\|}$

$= 50 \times \frac{(6, 8, -10)}{\sqrt{(6)^2+(8)^2+(-10)^2}}$

$= \left(15\sqrt{2}, 20\sqrt{2}, -25\sqrt{2}\right)$

$\therefore \overrightarrow{M_o} = \overrightarrow{OA} \times \overrightarrow{F}$

$= (0, 0, 10) \times \left(15\sqrt{2}, 20\sqrt{2}, -25\sqrt{2}\right)$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 10 \\ 15\sqrt{2} & 20\sqrt{2} & -25\sqrt{2} \end{vmatrix}$

$= -200\sqrt{2}\,\hat{i} + 150\sqrt{2}\,\hat{j}$

Model 5

1

(1) Magnitudes of limiting static friction force and the normal reaction.

(2) $-11\hat{i} - 17\hat{j} + \hat{k}$ $\quad$ (3) $10\frac{1}{3}$ $\quad$ (4) 2

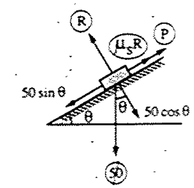
(5) • The algebraic sum of the components of the forces in any two orthogonal directions in the plane of the forces must be zero.

• The algebraic sum of the moments of all the forces about one point in the plane of forces must be zero.

(6) The point of suspension.

2

[a] When the body is about to move downwards (the least force)



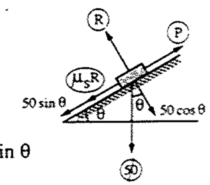
$\therefore P + \mu_s R = 50 \sin\theta$

$, R = 50 \cos\theta$

$\therefore 10 + \mu_s \times 50\cos\theta = 50\sin\theta$

$\therefore \mu_s = \frac{50\sin\theta - 10}{50\cos\theta}$ $\quad$ (1)

and when the body is about to move up. (The greatest force)



$\therefore P = \mu_s R + 50\sin\theta$

$, R = 50\cos\theta$

$40 = \mu_s \times 50\cos\theta + 50\sin\theta$

$\therefore \mu_s = \frac{40 - 50\sin\theta}{50\cos\theta}$ $\quad$ (2)

From (1) , (2) : $\frac{50\sin\theta - 10}{50\cos\theta} = \frac{40 - 50\sin\theta}{50\cos\theta}$

$\therefore 100\sin\theta = 50$ $\quad\quad \therefore \sin\theta = \frac{1}{2}$

$\theta = 30°$, $\quad\quad \therefore \mu_s = \frac{\sqrt{3}}{5}$

[b] $M_A = \pm 200$ newton.cm.

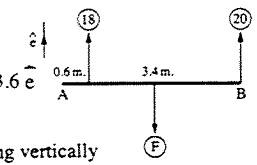
$\therefore -F\cos 30° \times 40 - F\sin 30° \times 5 = -200$

$\therefore F(40\cos 30° + 5\sin 30°) = 200$

$\therefore F \simeq 5.38$ newton.

3

[a]



$\because 20\,\overrightarrow{e} + 18\,\overrightarrow{e} + \overrightarrow{F} = 13.6\,\overrightarrow{e}$

$\therefore \overrightarrow{F} = -24.4\,\overrightarrow{e}$

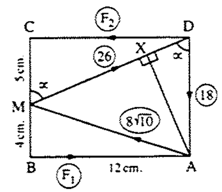
∴ F = 24.4 kg.wt. acting vertically downwards and at a distance X from "A"

, ∵ the sum of moments of the forces about A = the sum of the moment of the resultant about A

$\therefore 20 \times 4 + 18 \times 0.6 - 24.4 \times X = 13.6 \times 3$

$\therefore X = 2.05$ metre

[b]



$\because MD = \sqrt{144 + 25} = 13$ cm.

$\because AM = \sqrt{144 + 16}$

$= \sqrt{160} = 4\sqrt{10}$

The force along $\overrightarrow{DA}$, $\overrightarrow{AM}$

, $\overrightarrow{MD}$ are in the same way round

$\because \frac{18}{9} = \frac{8\sqrt{10}}{4\sqrt{10}} = \frac{26}{13} = 2$

∴ The system is equivalent to a couple its algebraic measure (M_1)

∵ The area of Δ AMD = $\frac{1}{2} \times 9 \times 12 = 54$ cm^2

$\therefore M_1 = -2 \times 54 \times 2 = -216$ newton.cm.

∵ The couple can be in equilibrium only with another couple.

∴ (F_1 , F_2) determine a couple its algebraic measure = 216 newton.cm.

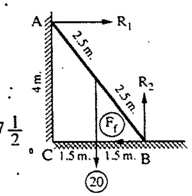
$\therefore F_1 = F_2 = F$ $\quad \therefore F \times 9 = 216$ $\quad \therefore F = 24$

$\therefore F_1 = 24$ newton , $F_2 = 24$ newton.

4

[a]

Let the ladder is in equilibrium :



$R_1 = F_r$, $R_2 = 20$

, M_B = zero

$\therefore 20 \times \frac{3}{2} - R_1 \times 4 = 0$

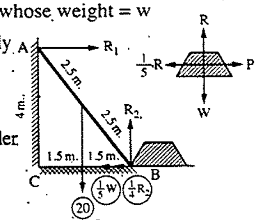
$\therefore 30 = 4R_1$ $\quad\quad \therefore R_1 = 7\frac{1}{2}$

∵ The limiting friction at B = $\mu R_2 = \frac{1}{4} \times 20 = 5$

∴ The ladder will not be in equilibrium in this case because F_r > the limiting friction.

After putting the body whose weight = w

• With respect to the body



$P = \frac{1}{5}R$, R = W

$\therefore P = \frac{1}{5}W$

• With respect to the ladder

$R_1 = \frac{1}{4}R_2 + \frac{1}{5}W$

$, R_2 = 20$

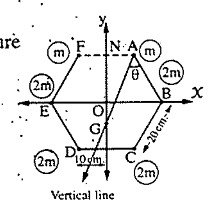
M_B = zero $\quad\quad \therefore 20 \times \frac{3}{2} - R_1 \times 4 = $ zero

$\therefore 30 = 4R_1$ $\quad\quad \therefore R_1 = 7\frac{1}{2}$

$\therefore 7\frac{1}{2} = \frac{1}{4} \times 20 + \frac{1}{5}W$ $\quad \therefore W = 12\frac{1}{2}$ kg.wt.

[b]

length of each side of the figure = 100 ÷ 5 = 20 cm.



let the mass of each of $\overline{AB}$, $\overline{BC}$

, $\overline{CD}$, $\overline{DE}$

, $\overline{EF}$ is 2 m and act at its midpoint

, by distributing the masses among the vertices of the hexagon.

	A	B	C	D	E	F
Mass	m	2 m	2 m	2 m	2 m	m
x	10	20	10	−10	−20	−10
y	$10\sqrt{3}$	0	$-10\sqrt{3}$	$-10\sqrt{3}$	0	$10\sqrt{3}$

$\therefore x_G = \frac{m \times 10 + 2m \times 20 + 2m \times 10 + 2m \times -10 + 2m \times -20 + m \times -10}{m + 2m + 2m + 2m + 2m + m}$

= zero

$, y_G = \frac{m \times 10\sqrt{3} + 2m \times -10\sqrt{3} + 2m \times -10\sqrt{3} + m \times 10\sqrt{3}}{m + 2m + 2m + 2m + 2m + m}$

$= -2\sqrt{3}$

$\therefore$ Centre of gravity $= (0 , -2\sqrt{3})$

, $\because$ centre of the hexagon $= (0 , 0)$

$\therefore$ Centre of gravity of the wire lies at a distance $2\sqrt{3}$ from the centre of the hexagon.

• At suspension from A from the figure we find that

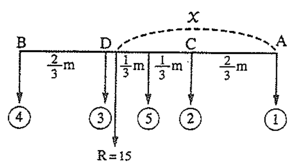
$\tan(\angle NAG) = \frac{12\sqrt{3}}{10} = \frac{6\sqrt{3}}{10}$

$\therefore m(\angle NAG) \simeq 64° 10\overset{\backprime}{8}$

$\therefore m(\angle \theta) = 120° - 64° 1\overset{\backprime}{8} = 55° 4\overset{\backprime}{2}$

5

[a]



Let the centre of gravity lies at a distance x metre from A , moment of the forces about A = moment of the resultant about A

$\therefore 2 \times \frac{2}{3} + 5 \times 1 + 3 \times \frac{4}{3} + 4 \times 2 = 15\,x$

$\therefore x = \frac{11}{9}$ metre

$\therefore$ The centre of gravity acts at a distance $\frac{11}{9}$ metre from A

[b] $\because \overrightarrow{F_1} = (2 , -1)$, $\overrightarrow{F_2} = (-2 , 1) = -(2 , -1) = -\overrightarrow{F_1}$

$\therefore \overrightarrow{F_1}$, $\overrightarrow{F_2}$ form a couple its algebric measure $(\overrightarrow{M})$

, $\because$ the moment of the couple is a constant , independent of the point about which we take the moments.

$\therefore$ The moment of the couple = the moment of the force $\overrightarrow{F_1}$ about the point B $= \overrightarrow{BA} \times \overrightarrow{F_1}$

$= (1 , 5) \times (2 , -1)$

$= -11\,\hat{k}$